JOURNAL OF GUIDANCE, CONTROL, AND DYNAMICS
Vol. 25, No. 4, July–August 2002

# Multiple-Model Adaptive Flight Control Scheme for Accommodation of Actuator Failures

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**A new parameterization for the modeling of control effector failures in flight control applications and a stable adaptive scheme for failure detection, identification, and accommodation is proposed. The failures include lock in place, hard over, and loss of effectiveness. It is shown that the resulting representation leads naturally to a multiple-model formulation of the corresponding control problem that can be solved using a multiple-model adaptive reconfigurable control approach. We derive stable multiple-model adaptive reconfigurable control algorithms for several cases of increasing complexity, including the most complex case, wherein one of the effectors undergoes lock-in-place or hard-over failure, and all others lose effectiveness. In all cases, the stability of the overall reconfigurable control system is demonstrated using multiple Lyapunov functions, extensions of the Lyapunov method, and the separation between identification and control arising in the context of indirect adaptive control. The approach is illustrated through numerical simulations of the F/A-18 aircraft during carrier landing.**

## I. Introduction

FAST and accurate flight control reconfiguration is of paramount importance for increasing aircraft survivability in the presence of severe subsystem failures and structural damage. As advocated by a number of researchers in the field of advanced flight control design, including the recent discussion from Ref. 1, adaptive control appears to be the most viable approach for achieving desired flight performance in the presence of large uncertainties, including abrupt, unknown variations arising due to severe subsystem failures. In addition, important information for the overall guidance and control system is that regarding the type and time of failure. However, whereas several adaptive reconfigurable flight control approaches have been proposed, for instance, those in Refs. 2–5, among many others, a detailed analysis of different failure parameterizations, corresponding adaptive control designs, and stability, robustness, and performance of the overall closed-loop reconfigurable control system is lacking in the existing literature. On the other hand, an approach for which some robustness properties can be determined is that from Refs. 6–8, based on neural network compensation of failures. This approach can not guarantee that the tracking error will tend to zero asymptotically and, in addition, is of the direct adaptive control type, hence failing to provide any information regarding the failure.

In this paper, we propose a new parameterization for a large class of failures encountered in flight control applications and develop stable multiple-model adaptive reconfigurable control algorithms to accommodate for the variation in aircraft dynamics due to such failures. It will be shown that the corresponding adaptive control algorithms are well suited to compensate for the effect of multiple simultaneous failures, even while maintaining the stability and robustness of the system.

Our focus is on different types of failures of control effectors, including the lock-in-place failure when the effector freezes in a certain position and does not respond to subsequent commands. As shown in Ref. 9, for an aircraft controlled by a full-state feedback controller, such a failure introduces both constant and state-dependent disturbances into the overall closed-loop system, and control reconfiguration is necessary for maintaining the stability and robustness of the system. As also shown in Ref. 10, adaptive control using a single model may not be adequate for achieving this task in the presence of failures of critical control effectors. This is because, in a particular flight regime, aircraft dynamics immediately after the failure may be very far from its nominal (no-failure) dynamics. Hence, a single-model-based adaptive controller may be too slow to bring the closed-loop system close to the new operating regime, which may result in unacceptably large transients. For this reason, in Ref. 10, a multiple-model-based reconfigurable control strategy was proposed. The corresponding controller is based on the control effector redundancy encountered in modern overactuated aircraft, and the concept of multiple models, switching and tuning (MMST) from Ref. 11 (Fig. 1).

The MMST concept is fairly general and is well suited for the case of plants with rapidly varying dynamics. If the plant dynamics in different operating regimes are described by different models, the MMST concept is implemented by using the outputs of the parallel estimators (identification models) $O_1, O_2, \ldots, O_N$ to find the model closest in some sense to the current plant dynamics and switch to the corresponding controller. In the context of reconfigurable control, each model in Fig. 1 represents a different failure scenario. Although the system is initiated with the controller for the no-failure case, in the case of failure, the objective is to design a suitable control reconfiguration algorithm to assure that the scheme switches to the controller corresponding to the model closest, in some sense, to the dynamics of the failed plant. The need for MMST in plants with rapidly switching dynamics can be summarized using Fig. 2.

As shown in Fig. 2, failure may cause the plant dynamics to switch abruptly from some nominal point $P_0$ in the parametric space, to the point $P$ corresponding to the failed plant. The top part of Fig. 2 illustrates the case when adaptation using a single model may be too slow to identify the new operating regime and reconfigure the controller. In such a case, placing several models in the parametric set, switching to the model close to the dynamics of the failed plant, and adapting from there can result in fast and accurate control reconfiguration.

This problem was studied in detail in Ref. 10, where both single and multiple adaptive controllers were used to achieve flight control reconfiguration in the presence of lock-in-place failure of a critical control effector. It was shown that, whereas a single-model-based adaptive controller yields unacceptable transients, a reconfigurable controller based on the MMST concept results in fast and accurate control reconfiguration. However, the overall scheme is well suited

Received 9 February 2001; revision received 17 September 2001; accepted for publication 1 October 2001. 

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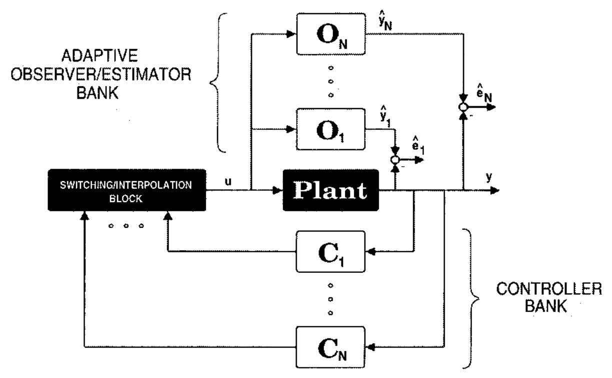


**Fig. 1 Structure of the MMST controller.**

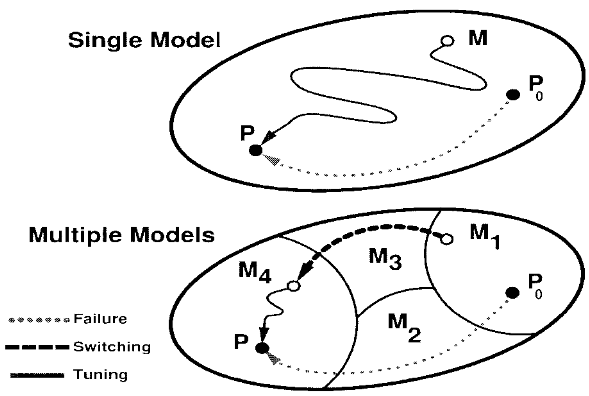


**Fig. 2 Single-model vs multiple-model adaptation.**

only for the lock-in-place type of control effector failures. In this paper, we extend the results from Ref. 12 and propose a reconfigurable control scheme that covers the case of a large number of control effector failures. The method is well suited for the case of a large number of failures and results in the number of models equal to the number of control effectors, plus an additional model for the no-failure case.

Our approach is based on the failure modeling, MMST concept, adaptive reconfigurable control design, multiple tentative Lyapunov functions, and extensions of the Lyapunov method. In Sec. II we state the reconfigurable control problem. New failure parameterization and the corresponding identification models are presented in Sec. III. Multiple-model adaptive control and corresponding proofs of stability for the cases without and with loss of effectiveness are given, respectively, in Secs. IV and V. Section VI contains extensions of the method. Section VII provides simulation results, and the conclusions are given in Sec. VIII, followed by Appendices A–I.

## II. Problem Statement

In this paper we will consider the following linearized aircraft model:

$$\dot{\boldsymbol{x}}_p = A_p \boldsymbol{x}_p + B_p \boldsymbol{u} \tag{1}$$

where $\boldsymbol{x}_p \in \mathbb{R}^n$ and $\boldsymbol{u} \in \mathbb{R}^m$ denote, respectively, the state and control input vectors; $A_p$ is an $(n \times n)$ matrix; and $B_p$ is an $(n \times m)$ matrix. We note that the matrices $A_p$ and $B_p$ are commonly gain scheduled with the flight condition and are generally state dependent.

Typical linearized aircraft models can be parameterized in the following form:

$$\dot{\boldsymbol{x}}_1 = \bar{A} \boldsymbol{x}_2 \tag{2}$$

$$\dot{\boldsymbol{x}}_2 = A\boldsymbol{x} + B\boldsymbol{u} \tag{3}$$

where $\boldsymbol{x} = [\boldsymbol{x}_1^T \;\; \boldsymbol{x}_2^T]^T$, $\boldsymbol{x}_1$ is an $(n-p)$ vector, $\boldsymbol{x}_2$ is a $p$ vector, $B$ is a $(p \times m)$ matrix, and matrices $\bar{A}$ and $A$ are of appropriate dimensions. The idea behind this parameterization is to decompose the model into a subsystem not directly affected by $\boldsymbol{u}$, such that the relative degree between $\boldsymbol{x}_1$ and $\boldsymbol{u}$ is equal to two, that is, the $\boldsymbol{x}_1$ subsystem, and a subsystem where the relative degree is equal to one, that is, the $\boldsymbol{x}_2$ subsystem. Also note that $\bar{A}$ is just a matrix of simple integrators. Such a parameterization arises from the flight control applications where linearized aircraft models already take this form. We also note that the elements of $\boldsymbol{x}$ and $\boldsymbol{u}$ are perturbed variables, that is, deviations from a trim condition.

Relative degree-two perturbed variables in typical linearized aircraft models include pitch angle $\theta$, altitude $h$, bank angle $\phi$, and yaw angle $\psi$, whereas relative degree-one perturbed variables are forward velocity $V$, pitch rate $q$, angle of attack $\alpha$, sideslip angle $\beta$, roll rate $p$, and yaw rate $r$.

We will also make the following assumption:

*Assumption 1.*

1) The state $\boldsymbol{x}(t)$ of the system is available at every instant.

2) The number of actuators exceeds the dimension of $\boldsymbol{x}_2$, that is, $m > p$.

3) Rank $(BB^T) = p$.

Assumption 1, part 1, is satisfied in the case of modern aircraft equipped with a large number of sensors and navigation filters, Assumption 1, part 3, is satisfied in a large number of flight regimes, and Assumption 1, part 2, corresponds to the case encountered in modern aircraft, where the number of control effectors exceeds the number of variables that can be directly affected by the controls.

In this paper, we will consider the case when the objective is to design a controller such that $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ follow the outputs of the following reference model:

$$\dot{\boldsymbol{x}}_{1m} = \bar{A}\boldsymbol{x}_{2m} \tag{4}$$

$$\dot{\boldsymbol{x}}_{2m} = A_m \boldsymbol{x}_m + B_m \boldsymbol{r} \tag{5}$$

where $\boldsymbol{x}_m = [\boldsymbol{x}_{1m}^T \;\; \boldsymbol{x}_{2m}^T]^T$, $\boldsymbol{x}_{1m}$ is an $(n-p)$ vector, $\boldsymbol{x}_{2m}$ is a $k$ vector, $A_m$ is asymptotically stable $(p \times p)$ matrix, $B_m$ is a $(p \times p)$ matrix, and $\boldsymbol{r}$ denotes a $p$ vector of bounded reference inputs such that $\dot{\boldsymbol{r}} \in \mathcal{L}^\infty$. Furthermore, the matrix $A_m$ is such that

$$\Lambda_0 = \begin{bmatrix} 0_{(n-k)\times k} & \bar{A} \\ & A_m \end{bmatrix}$$

is asymptotically stable.

Based on Assumptions 1, the state vector is measurable, $BB^T$ is full rank, and there is an excess of control inputs with respect to the controlled outputs. Hence, the control objective in the ideal case that is, the case without control effector failures, can be achieved using the following control law:

$$\boldsymbol{u} = W^{-1}B^T(BW^{-1}B^T)^{-1}\{-A\boldsymbol{x} + \Lambda(\boldsymbol{x} - \boldsymbol{x}_m) + A_m\boldsymbol{x}_m + B_m\boldsymbol{r}\} \tag{6}$$

where $\Lambda$ is such that the matrix $\Lambda_0 = [\bar{A}^T \;\; \Lambda^T]^T$ is asymptotically stable. This control law is based on the pseudoinverse control algorithm from Appendix A and is also referred to as the inverse dynamics control law (IDCL).

Let $\boldsymbol{e}_1 = \boldsymbol{x}_1 - \boldsymbol{x}_{1m}$ and $\boldsymbol{e}_2 = \boldsymbol{x}_2 - \boldsymbol{x}_{2m}$ denote the output error vectors, and $\boldsymbol{e} = [\boldsymbol{e}_1^T \;\; \boldsymbol{e}_2^T]^T$. From Eqs. (2–6), we obtain the following error model:

$$\dot{\boldsymbol{e}} = \Lambda_0 \boldsymbol{e} \tag{7}$$

This system is exponentially stable because $\Lambda_0$ is asymptotically stable. Hence, the control law (6) achieves the objective, that is, forces $\boldsymbol{x}_1(t)$ and $\boldsymbol{x}_2(t)$ to follow, respectively, $\boldsymbol{x}_{1m}(t)$ and $\boldsymbol{x}_{2m}(t)$ asymptotically in the case without failures.

However, it was shown in Ref. 9 that such a control law, in general, can not stabilize the closed-loop system in the case of control effector lock in place and that control reconfiguration is necessary to achieve the objective. Hence, we consider the following control objective.

The control objective begins with the design of a control law $\boldsymbol{u}(t)$ for the plant (2) and (3) such that all signals in the system are bounded and, in addition,

$$\lim_{t\to\infty}[\boldsymbol{x}_1(t) - \boldsymbol{x}_{1m}(t)] = \lim_{t\to\infty}[\boldsymbol{x}_2(t) - \boldsymbol{x}_{2m}(t)] = 0 \tag{8}$$

in the presence of different types of control effector failures.

To solve this problem, we will first consider the parameterization of different control effector failures. Our objective is to derive suitable parameterization and corresponding reconfigurable control algorithms to achieve the control objective in the presence of a large class of failures. This is discussed next.

## III. Parameterization of Control Effector Failures

Common control effector failures include 1) freezing or lock in place, 2) hard over, and 3) loss of effectiveness. In the case of freezing, the effector gets locked in a certain position and does not respond to subsequent commands; hard over is a special case of freezing when the effector locks at its position saturation limit. These two types of failures are particularly severe in the case of critical control effectors because, in the case of a full-state feedback baseline controller, they generate both constant and state-dependent disturbances in the overall closed-loop system. As shown in Ref. 9, the lack of compensation for such disturbances may lead to substantial performance deterioration, instability of the overall system, and loss of the aircraft. The case of loss of effectiveness corresponds to a slow or sudden decrease in the control effector gain so that the same control signal results in a smaller deflection of the effector as compared to that achieved in the case with no failure.

Our objective is to develop a failure parameterization model that includes all of the preceding cases.

Let us, for simplicity, assume that the transfer functions of the actuators is equal to one, and let

$$\tilde{\boldsymbol{u}} = B\boldsymbol{u} = BK\boldsymbol{u}_c = [b_1k_1u_{c1} \quad b_2k_2u_{c2} \quad \cdots \quad b_mk_mu_{cm}] \tag{9}$$

where $\boldsymbol{u}_c$ denotes the vector of signals generated by the controller, $K = \mathrm{diag}[k_1 \ \ k_2 \ \ \cdots \ \ k_m]$ and $\boldsymbol{b}_j$ denotes the $j$th column of $B$. This relationship is represented graphically in Fig. 3.

It is clear that in the case with no failures, $k_i = 1$, and $u_i = u_{ci}$, $i = 1, 2, \ldots, m$.

In this paper, we propose the following parameterization of different types of control effector failures, specifically, the no-failure case, the loss of effectiveness, the lock-in-place failure, and, the hard-over failure, respectively:

$$u_i(t) = \begin{cases} u_{ci}(t), & k_i(t) = 1, & \text{for all } t \geq t_0 \\ k_i(t)u_{ci}(t), & 0 < \epsilon_i \leq k_i(t) < 1, & \text{for all } t \geq t_{Fi} \\ u_{ci}(t_{Fi}), & k_i(t) = 0, & \text{for all } t \geq t_{Fi} \\ (u_i)_{\min} \text{ or } (u_i)_{\max}, & k_i(t) = 0 & \text{for all } t \geq t_{Fi} \end{cases}$$

where $t_{Fi}$ denotes the time instant of failure of the $i$th effector, $k_i$ denotes its effectiveness coefficient such that $k_i \in [\epsilon_i, 1]$, and $\epsilon_i > 0$ denotes its minimum effectiveness.

*Comments:*

1) The preceding parameterization is given in terms of the signals $u_i(t)$ and (possibly) time-varying gains $k_i(t)$. As we will show in the following sections, such a parameterization enables us to estimate $u_i$ and $k_i$ separately.

2) The preceding parameterization covers several different cases of control effector failures. In the case of loss of effectiveness, we assumed that the effector effectiveness cannot be less than some minimum effectiveness $\epsilon_i > 0$. Otherwise, in the case when the effectiveness of all effectors is zero, the aircraft becomes uncontrollable and inevitably crashes. In this paper, we will consider the case when all effectors can lose effectiveness up to $\epsilon_i > 0$ and show that with our adaptive reconfigurable controller, the overall system will still be stable and the control objective will be achieved. Also, we will show that our approach is well suited for the case when one of the effectors fails (lock in place or hard over), whereas all other effectors loose effectiveness.

3) As will be shown in the following section, an important element in our design is that, in the case of lock-in-place and hard-over failures, $\dot{u}_i(t) = 0$, $\forall t \geq t_{Fi}$.

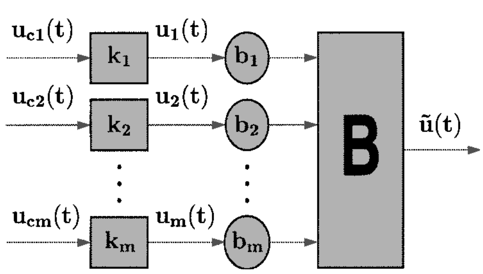


**Fig. 3 Relationship between $u_c$ and $u$.**

We will now show that the preceding parameterization leads naturally to a multiple-model formulation of the corresponding reconfigurable control problem.

Using expression (9), we first rewrite the plant equation (2) and (3) in the form

$$\dot{\boldsymbol{x}}_1 = \bar{A}\boldsymbol{x}_2 \tag{10}$$

$$\dot{\boldsymbol{x}}_2 = A\boldsymbol{x} + \boldsymbol{b}_1k_1u_{c1} + \boldsymbol{b}_2k_2u_{c2} + \cdots + \boldsymbol{b}_mk_mu_{cm} \tag{11}$$

This is the model of the plant in the case with no failures and can be expressed in a compact form as

$$\dot{\boldsymbol{x}}_1 = \bar{A}\boldsymbol{x}_2 \tag{12}$$

$$\dot{\boldsymbol{x}}_2 = A\boldsymbol{x} + BK\boldsymbol{u}_c \tag{13}$$

where $K = \mathrm{diag}[k_1 \ \ k_2 \ \ \cdots \ \ k_m]$.

Using the preceding parameterization of $u_i$, we next focus on the case of failures of control effectors. It is seen that each case of failure can be modeled by a different model, resulting in the total of $m$ equations of the form

$$\dot{\boldsymbol{x}}_{1i} = \bar{A}\boldsymbol{x}_{2i} \tag{14}$$

$$\dot{\boldsymbol{x}}_{2i} = A\boldsymbol{x} + \bar{B}_iK\boldsymbol{u}_c + \boldsymbol{b}_i\bar{u}_i, \qquad i = 1, 2, \ldots, m \tag{15}$$

where $\bar{u}_i$ can assume the following values: 1) $u_{ci}(t_{Fi})$ (lock in place), or 2) $(u_i)_{\min}$ or $(u_i)_{\max}$ (hard over). Matrices $\bar{B}_i$ from the preceding expression are of the form

$$\begin{aligned} \bar{B}_1 &= [\boldsymbol{0} \quad \boldsymbol{b}_2 \quad \boldsymbol{b}_3 \quad \ldots \quad \boldsymbol{b}_{m-1} \quad \boldsymbol{b}_m] \\ \bar{B}_2 &= [\boldsymbol{b}_1 \quad \boldsymbol{0} \quad \boldsymbol{b}_3 \quad \ldots \quad \boldsymbol{b}_{m-1} \quad \boldsymbol{b}_m] \\ &\vdots \\ \bar{B}_{m-1} &= [\boldsymbol{b}_1 \quad \boldsymbol{b}_2 \quad \boldsymbol{b}_3 \quad \ldots \quad \boldsymbol{0} \quad \boldsymbol{b}_m] \\ \bar{B}_m &= [\boldsymbol{b}_1 \quad \boldsymbol{b}_2 \quad \boldsymbol{b}_3 \quad \ldots \quad \boldsymbol{b}_{m-1} \quad \boldsymbol{0}] \end{aligned}$$

where $\boldsymbol{0}$ is an $m$ vector with zero elements. It is seen that each failure is modeled by removing the corresponding column of matrix $B$ and adding the term $\boldsymbol{b}_i\bar{u}_i$.

The corresponding identification models can be chosen in the following form:

$$\dot{\hat{\boldsymbol{x}}}_{10} = \bar{A}\hat{\boldsymbol{x}}_{20} \tag{16}$$

$$\dot{\hat{\boldsymbol{x}}}_{20} = \Lambda\hat{\boldsymbol{e}}_0 + A\boldsymbol{x} + B\hat{K}_0\boldsymbol{u}_c \tag{17}$$

$$\dot{\hat{\boldsymbol{x}}}_{1i} = \bar{A}\hat{\boldsymbol{x}}_{2i} \tag{18}$$

$$\dot{\hat{\boldsymbol{x}}}_{2i} = \Lambda\hat{\boldsymbol{e}}_i + A\boldsymbol{x} + \bar{B}_i\hat{K}_i\boldsymbol{u}_c + \boldsymbol{b}_i\hat{u}_i, \qquad i = 1, 2, \ldots, m \tag{19}$$

where $\hat{\boldsymbol{e}}_i = \hat{\boldsymbol{x}}_i - \boldsymbol{x}$, $i = 0, 1, 2, \ldots, m$; $\hat{\boldsymbol{e}}_i = [\hat{\boldsymbol{e}}_{1i}^T \ \ \hat{\boldsymbol{e}}_{2i}^T]^T$; $\hat{\boldsymbol{x}}_i = [\hat{\boldsymbol{x}}_{1i}^T \ \ \hat{\boldsymbol{x}}_{2i}^T]^T$; and $\hat{K}_i$ and $\hat{u}_i$ denote, respectively, the estimates of $K$ and $u_i$. In the preceding equations, subscript 0 corresponds to the no-failure case, and $\Lambda$ is chosen such that the matrix

$$\Lambda_0 = \begin{bmatrix} 0_{(n-k)\times k} & \bar{A} \\ & \Lambda \end{bmatrix} \tag{20}$$

is asymptotically stable.

*Comment:* One of the first questions that arise after studying the preceding identification models is as to whether it would be possible to estimate all of the failures using a single identification model. However, there are several problems with such an approach, even in the case when $K = I_{m\times m}$. Because we do not know in advance which effector will fail and when, and because the signals $u_i$ are, in general, time varying, such an approach would lead to simultaneous identification of $m$ time-varying signals. Hence, even in the case with no failures, we can not guarantee that the identification error will tend to zero. Another possibility is to use only two models: one for the no-failure case and the other one for estimating the failure. However, the problem again reduces to that of estimating $m$ time-varying signals online. Even if one or more effectors fail in the sense that $\dot{u}_i(t) = 0$ for all $t \geq t_{Fi}$, the remaining effectors will generate forcing terms due to the derivatives $\dot{u}_j(t)$, so that the most that we can show is that the resulting identification error is bounded, but not that it tends to zero asymptotically. As we will show in the following sections, the approach based on $m+1$ identification models removes these disadvantages.

Our objective is to design control algorithms corresponding to the preceding identification models and devise a suitable strategy for switching among such controllers so that the control objective is met in the presence of the failures just discussed. The baseline control strategy is discussed in the following section.

## IV. Baseline Reconfigurable Controller

To derive the reconfigurable control laws for the preceding cases, we first rewrite the baseline control law (6) in the form

$$\boldsymbol{u}_c = W^{-1}B^T(BW^{-1}B^T)^{-1}\boldsymbol{\eta} \tag{21}$$

where

$$\boldsymbol{\eta} = -A\boldsymbol{x} + \Lambda(\boldsymbol{x} - \boldsymbol{x}_m) + A_m\boldsymbol{x}_m + B_m\boldsymbol{r} \tag{22}$$

Based on the preceding failure models and the approach from Ref. 9, we design $m+1$ controllers of the form

$$\boldsymbol{u}_{ci} = K^{-1}W^{-1}B^T(BW^{-1}B^T)^{-1}\Theta_i[\boldsymbol{\eta} - \boldsymbol{b}_i\bar{u}_i]\qquad i = 0, 1, 2, \ldots, m \tag{23}$$

where $\Theta_i \in \mathbb{R}^{k\times k}$. It is clear that, in the case with no failures, $K = I_{m\times m}$, $\Theta_0 = I_{k\times k}$, and $\bar{u}_0 = 0$. In the case of failures,

$$\Theta_i = BW^{-1}B^T\left[\bar{B}_iW^{-1}\bar{B}_i^T\right]^{-1}, \qquad i = 1, 2, \ldots, m \tag{24}$$

As shown in Ref. 9, such controllers achieve the objective in the presence of control effector freezing. We will extend this approach to the case of other unknown failures as well (hard over and loss of effectiveness).

The preceding controllers are designed for the ideal case, that is, assuming that the type of failure, that is, the value of $\bar{\boldsymbol{u}}_i$, and the loss-of-effectiveness coefficients, that is, the matrix $K$, are known. The corresponding multiple reconfigurable adaptive controllers are based on online estimation of $K_i$ and $\bar{u}_i$ and are of the form

$$\boldsymbol{u}_{ci} = \hat{K}_i^{-1}W^{-1}B^T(BW^{-1}B^T)^{-1}[\Theta_i(\eta - b_i\hat{u}_i)]\qquad i = 0, 1, 2, \ldots, m \tag{25}$$

Our next objective is to devise a suitable strategy for switching among these controllers so that the control objective is achieved in the presence of the failures just discussed.

## V. Multiple-Model Adaptive Controller

The design of a reconfigurable control strategy that is based on switching among the given controllers, and that results in a stable overall system, is a complex task. Hence, we will divide our analysis into three parts: 1) In the first part, we will consider the case without loss of effectiveness covering the cases of lock-in-place and hard-over failures. 2) In the second part, we will consider the loss of effectiveness type of failure only. 3) In the third part, we will design an adaptive reconfigurable controller for the case when one of the effectors undergoes a lock-in-place or hard-over failure and all other effectors loose effectiveness.

### A. Case Without Loss of Effectiveness

We will start our analysis with the case when the effectors can undergo only lock-in-place or hard-over failures. In such a case, $K = I_{m\times m}$. Based on Eqs. (16–19) with $\hat{K}_i = I_{m\times m}$, we next build $m+1$ identification models for this case. The models are of the form

$$\dot{\hat{\boldsymbol{x}}}_{10} = \bar{A}\hat{\boldsymbol{x}}_{20} \tag{26}$$

$$\dot{\hat{\boldsymbol{x}}}_{20} = \Lambda\hat{\boldsymbol{e}}_0 + A\boldsymbol{x} + B\boldsymbol{u}_c \tag{27}$$

$$\dot{\hat{\boldsymbol{x}}}_{1i} = \bar{A}\hat{\boldsymbol{x}}_{2i} \tag{28}$$

$$\dot{\hat{\boldsymbol{x}}}_{2i} = \Lambda\hat{\boldsymbol{e}}_i + A\boldsymbol{x} + \bar{B}_i\boldsymbol{u}_c + \boldsymbol{b}_i\hat{u}_i, \qquad i = 1, 2, \ldots, m \tag{29}$$

The corresponding controllers are based on Eq. (25) in the case when $\hat{K}_i = I_{m\times m}$ and are of the form

$$\boldsymbol{u}_{ci} = W^{-1}B^T(BW^{-1}B^T)^{-1}\Theta_i[\eta - b_i\hat{u}_i]\qquad i = 0, 1, 2, \ldots, m \tag{30}$$

where $\Theta_0 = I_{k\times k}$, remaining matrices $\Theta_i$ are defined in Eq. (24), $\hat{u}_i$ denote the estimates of $\bar{u}_i$, and $\hat{u}_0 \equiv 0$.

To devise a suitable strategy for switching among these controllers, we will first consider the case when the system has the information about the failure. Although this is not a realistic assumption, it serves as a basis for the adaptive reconfigurable control design in the case when this information is not available.

#### 1. *Case of Known Failure*

We first substitute the controllers (30) into the corresponding identification models (26–29) to obtain

$$\dot{\hat{\boldsymbol{e}}}_{mi} = \Lambda_0\hat{\boldsymbol{e}}_{mi}, \qquad i = 0, 1, 2, \ldots, m \tag{31}$$

where $\hat{\boldsymbol{e}}_{mi} = \hat{\boldsymbol{x}}_i - \boldsymbol{x}_m$. Because $\Lambda_0$ is asymptotically stable, the given systems are exponentially stable, which implies that, for every $i$, $\lim_{t\to\infty}\hat{\boldsymbol{e}}_{mi}(t) = 0$. The fact that with the $i$th controller the $i$th observer behaves asymptotically as the reference model will be used in our adaptive control design.

We next consider the two possible cases: 1) case with no failure and 2) case when there is a failure of the $j$th effector at instant $t_{Fj}$ (lock in place or hard over) and this information is available to the controller. Because the information about the failure is available, we will use $\boldsymbol{u}_{co}$ in case 1 and $\boldsymbol{u}_{cj}$ in case 2.

*a. Case 1 (no failure):* From Eqs. (26), (27), (12), and (13), for the case when there is no loss of effectiveness, that is, when $K = I_{m\times m}$, we have

$$\dot{\hat{\boldsymbol{e}}}_0 = \Lambda_0\hat{\boldsymbol{e}}_0 \tag{32}$$

Because in this case $\boldsymbol{u}_{co}$ is used, from Eq. (31) we also have

$$\dot{\hat{\boldsymbol{e}}}_{m0} = \Lambda_0\hat{\boldsymbol{e}}_{m0} \tag{33}$$

From Eqs. (28), (29), (14), and (15) for $K = I_{m\times m}$, we now have

$$\dot{\hat{\boldsymbol{e}}}_{1i} = \bar{A}\hat{\boldsymbol{e}}_{2i} \tag{34}$$

$$\dot{\hat{\boldsymbol{e}}}_{2i} = \Lambda\hat{\boldsymbol{e}}_i + b_i\phi_i, \qquad i = 1, 2, \ldots, m \tag{35}$$

where $\phi_i = \hat{u}_i - u_i$. We note that the signals $u_i$ are, in general, not measurable. Hence, both $u_i$ and $\dot{u}_i$ will be assumed unknown.

Let $P$ denote a symmetric positive-definite solution of the Lyapunov matrix equation

$$\Lambda_0^TP + P\Lambda_0 = -Q \tag{36}$$

where $Q = Q^T > 0$. We next consider the following theorem.

*Theorem 1:* If there are no control effector failures, if the controller $\boldsymbol{u}_{co}$ is used for all time, and if the estimates $\hat{u}_i$ are adjusted using the following adaptive algorithms,

$$\dot{\hat{u}}_i = \operatorname*{Proj}_{[(u_i)_{\min},(u_i)_{\max}]}\left\{-\gamma_i\hat{\boldsymbol{e}}_i^TP\boldsymbol{b}_i\right\}$$

$$\hat{u}_i(0) \in [(u_i)_{\min}, (u_i)_{\max}], \qquad i = 1, 2, \ldots, m \tag{37}$$

where $\gamma_i > 0$ denote adaptive gains, then 1) all signals in the system are bounded and 2) $\lim_{t\to\infty}[\boldsymbol{x}(t) - \boldsymbol{x}_m(t)] = 0$.

*Proof:* The proof is given in Appendix A.

*b. Case 2 (failure of the jth effector):* In this case the plant equation is of the form

$$\dot{\boldsymbol{x}}_1 = \bar{A}\boldsymbol{x}_2 \tag{38}$$

$$\dot{\boldsymbol{x}}_2 = A\boldsymbol{x} + \boldsymbol{b}_1 u_{c1} + \boldsymbol{b}_2 u_{c2} + \cdots + \boldsymbol{b}_j \bar{u}_{cj} + \cdots + \boldsymbol{b}_m u_{cm} \tag{39}$$

where $\bar{u}_{cj} = u_{cj}(t_{Fj})$ is a constant. From the preceding equations and Eqs. (26) and (27), we have

$$\dot{\hat{\boldsymbol{e}}}_{10} = \bar{A}\hat{\boldsymbol{e}}_{20} \tag{40}$$

$$\dot{\hat{\boldsymbol{e}}}_{20} = \Lambda_0 \hat{\boldsymbol{e}}_0 + b_j(u_{cj} - \bar{u}_{cj}) \tag{41}$$

Furthermore, from Eqs. (38), (39), (28), and (29), we have

$$\dot{\hat{\boldsymbol{e}}}_{1j} = \bar{A}\hat{\boldsymbol{e}}_{2j} \tag{42}$$

$$\dot{\hat{\boldsymbol{e}}}_{2j} = \Lambda_0 \hat{\boldsymbol{e}}_j + b_j \phi_j \tag{43}$$

Because, in this case, $\boldsymbol{u}_j$ is used, from Eq. (31) we have

$$\dot{\hat{\boldsymbol{e}}}_{mj} = \Lambda_0 \hat{\boldsymbol{e}}_{mj} \tag{44}$$

We now focus on the remaining $m - 1$ identification models. From Eqs. (28) and (29) and Eqs. (38) and (39), we now have

$$\dot{\hat{\boldsymbol{e}}}_{1i} = \bar{A}\hat{\boldsymbol{e}}_{2i} \tag{45}$$

$$\dot{\hat{\boldsymbol{e}}}_{2i} = \Lambda \hat{\boldsymbol{e}}_i + b_i \phi_i, \qquad i = 1, 2, \ldots, m, \quad i \neq j \tag{46}$$

We also consider the following theorem.

*Theorem 2:* If there is a failure of the $j$th control effector, if the controller $\boldsymbol{u}_{cj}(t)$ is used for all $t \geq t_{Fj}$, and if the estimates $\hat{u}_i$ are adjusted using the adaptive algorithms (37), then 1) all signals in the system are bounded, 2) $\lim_{t\to\infty}[\boldsymbol{x}(t) - \boldsymbol{x}_m(t)] = 0$, 3) $\lim_{t\to\infty}\phi_j(t) = 0$.

*Proof:* The proof is given in Appendix B.

Based on Theorems 1 and 2, we next consider Theorem 3.

*Theorem 3:* If the system stays at $\boldsymbol{u}_{co}$ in the case without failures, or is switched at $t = t_{Fj}$ to the controller $\boldsymbol{u}_{cj}$ in the case of known failure, adaptive algorithms (37) assure that all signals are bounded and, in addition, $\lim_{t\to\infty}[\boldsymbol{x}(t) - \boldsymbol{x}_m(t)] = 0$.

*Proof:* The proof follows directly from the proofs of Theorems 1 and 2, Appendices A and B. □

Thus far we have assumed that the time instant and type of failure are known. It is seen that, when the current controller is $\boldsymbol{u}_{co}$ in the no-failure case, or $\boldsymbol{u}_{cj}$ in the case of lock-in-place or hard-over failure, the overall system will be stable, and the control objective will be met. We will use this analysis as a basis for our switching reconfigurable control strategy. This is discussed in the following section.

### 2. *Multiple-Model Adaptive Controller*

In this section, we will extend the earlier results to the case of unknown failure. This problem will be solved using multiple-model adaptive control.

*a. Overall controller:* The overall controller consists of the identification models (26–29), reference model (4) and (5), and controllers (30), where the estimates $\hat{u}_i$ are adjusted using Eq. (37). The objective is to devise a suitable strategy for switching among these controllers to achieve the control objective in the presence of lock-in-place and hard-over types of control effector failures.

*b. Switching scheme:* The switching scheme used in this paper is similar to that from Ref. 11 and is based on the following performance indices:

$$I_j(t) = c_1\|\hat{\boldsymbol{e}}_j(t)\|^2 + \frac{c_2}{c_3 t + 1}\int_{t_0}^{t}\|\hat{\boldsymbol{e}}_j(\tau)\|^2\,\mathrm{d}\tau$$

$$j = 0, 1, 2, \ldots, m \tag{47}$$

where $\hat{\boldsymbol{e}}_j = \hat{\boldsymbol{x}}_j - \boldsymbol{x}$ and $c_i > 0, i = 1, 2, 3$. The scheme is started with $\boldsymbol{u}_{co}$ and is implemented by calculating and comparing the preceding indices every $t_s$ instants and finding their minimum. The scheme switches to (or stays at) the corresponding controller.

We will next prove that such a strategy results in a stable overall system in which the control objective is accomplished.

*Theorem 4:* The preceding switching scheme assures the boundedness of all signals in the system (12) and (13) (with $K = I_{m\times m}$), (26–29), (4), (5), (30), and (37), and guarantees that $\lim_{t\to\infty}[\boldsymbol{x}(t) - \boldsymbol{x}_m(t)] = 0$.

*Proof:* The proof is given in Appendix C.

*Comments:*

1) Estimates $\hat{u}_i$ can be thought of as failure indicators. In the case of lock-in-place or hard-over failure of the $j$th effector, the derivative of $u_{cj}$ will be zero and we will have $u_{cj}(t) = \bar{u}_j$ for all $t \geq t_{Fj}$. Because a constant is sufficiently persistently exciting in one-dimensional parameter space, that each identification model contains only one adjustable parameter results in convergence of the error $\hat{u}_j(t) - u_j$ to zero. Hence, besides achieving the control objective in the presence of unknown failures, accurate and efficient failure detection and identification (FDI) is also achieved.

2) All proofs are based on the separation between identification and control arising in the context of indirect adaptive control. As discussed extensively by several authors in the 1970s and 1980s (see, for example, Ref. 13), the certainty-equivalence principle is based on generating the plant parameter estimates by a suitably chosen identification model and using these estimates in the control law at every instant. Such a strategy was shown to enable the design of an identification model that is (to a large extent) independent of the control law. In particular, in the case of continuous-time systems, as long as the control input and state are bounded, the properties of the identification model (boundedness of the signals and asymptotic convergence of the identification and parameter errors to zero) can be established independently of the nature of the control law.

3) The separation between identification and control in the context of multiple identification models enabled us to use multiple tentative Lyapunov functions and establish separately the properties of each identification model.

4) The analysis of both open-loop and closed-loop behavior of each observer using multiple tentative Lyapunov functions enabled us to demonstrate the following: a) With the adaptive laws (37), all identification errors will be bounded, regardless of the current controller. This can be shown by the reconfigurable control problem being formulated as that of indirect adaptive control in the presence of time-varying parameters, where their estimates are bounded by the choice of adaptive algorithms. b) If at least one of the models coincides with the current plant dynamics, say the $l$th model, then $\hat{\boldsymbol{e}}_l \in \mathcal{L}^2 \cap \mathcal{L}^\infty$, which holds independently of the current controller. c) The closed-loop dynamics of the identification model corresponding to the current controller will asymptotically approach that of the reference model, which, after some analysis, implies that $\boldsymbol{x}$ and $u_{ci}$ are bounded. d) Conditions b and c imply that $\hat{\boldsymbol{e}}_l(t)$ will tend to zero asymptotically. e) After some analysis, we can also show that the error $\boldsymbol{e}_{ml}(t) = \hat{\boldsymbol{x}}_{ml}(t) - \boldsymbol{x}(t)$ will also tend to zero asymptotically. f) Conditions d and e imply that the tracking objective is achieved, that is, that $\lim_{t\to\infty}[\boldsymbol{x}(t) - \boldsymbol{x}_m(t)] = 0$.

5) From condition d of comment 4, it follows that, even with a wrong controller, $\lim_{t\to\infty}\hat{\boldsymbol{e}}_l(t) = 0$. Because all other identification models will have forcing terms, their performance indices will be nonzero. Because of the term $c_2/(c_3 t + 1)$ and because $\hat{\boldsymbol{e}}_l(t)$ tends to zero, we can conclude that $\lim_{t\to\infty} I_l(t) = 0$. Hence, as shown in the preceding theorem, the scheme is guaranteed to switch to the right controller.

## B. Loss of Effectiveness Type of Failure

In this section, we will assume that all effectors can undergo loss of effectiveness and that there are no other failures.

We first consider the following assumption.

*Assumption 2:* We assume that $\dot{k}_i \in \mathcal{L}^2 \cap \mathcal{L}^\infty$ and $\ddot{k}_i \in \mathcal{L}^\infty$ for all $i = 1, 2, \ldots, m$.

We are essentially assuming that the loss of effectiveness can occur either as a slow or abrupt variation of $k_i(t)$ and that each $k_i(t)$ eventually settles at the new value.

In this case, the identification model is of the form

$$\dot{\hat{x}}_{10} = \bar{A}\hat{x}_{20} \tag{48}$$

$$\dot{\hat{x}}_{20} = \Lambda\hat{x}_0 + Ax + B\hat{K}u_c \tag{49}$$

where the elements of the diagonal matrix $\hat{K}$ are estimates $\hat{k}_i$ of $k_i$, $i = 1, 2, \ldots, m$.

In this case, $u_c = u_{co}$, so that

$$u_c = \hat{K}^{-1}W^{-1}B^T(BW^{-1}B^T)^{-1}\eta \tag{50}$$

where $\eta$ is defined in Eq. (22).

Let $\bar{P} = P[0 \;\; B^T]^T$, where $P$ is defined in Eq. (36). Adaptive algorithms are chosen in the form

$$\dot{\hat{k}}_i = \underset{[\epsilon_i, 1]}{\text{Proj}}\left\{-\gamma_{ki}\hat{e}_0^T\bar{p}_i u_{ci}\right\}, \qquad \hat{k}_i(0) \in [\epsilon_i, 0] \tag{51}$$

where $\gamma_{ki} > 0$ and $\bar{p}_i$ denotes the $i$th column of $\bar{P}$.

We next consider the following proposition.

*Proposition 1:* Adaptive algorithms (51) assure that

$$\phi_k^T\Gamma_k^{-1}\dot{\phi}_k \le -\phi_k^T U_c\bar{P}^T\hat{e}_0 - \phi_k^T\Gamma_k^{-1}\dot{k} \tag{52}$$

*Proof:* After premultiplying the expressions from Eq. (51) for every $i$ with the corresponding $\hat{k}_i/\gamma_{ki}$, and adding the resulting equations, we obtain

$$\hat{k}^T\Gamma_k^{-1}\dot{\hat{k}} = \sum_{i=1}^{m}\underset{[\epsilon_i, 1]}{\text{Proj}}\left\{-\hat{k}_i\hat{e}_0^T\bar{p}_i u_{ci}\right\} \tag{53}$$

Using the properties of the adaptive algorithms with projection, we then have

$$\hat{k}^T\Gamma_k^{-1}\dot{\hat{k}} \le -\hat{k}^T U_c\bar{P}^T\hat{e}_0 \tag{54}$$

Because $\dot{\phi}_{ki} = \dot{\hat{k}}_i - \dot{k}_i$, Eq. (52) holds. □

We next consider the following theorem.

*Theorem 5:* Adaptive algorithms (51) assure that all signals in the system (12), (13), (48), (49), (50), and (22) are bounded and that, in addition, $\lim_{t\to\infty}[x(t) - x_m(t)] = 0$.

*Proof:* The proof is given in Appendix D.

*Comment:* In this case, a single identification model was used. Multiple-model control design in the case of simultaneous failures and loss of effectiveness is discussed in the following section.

### C. Integrated Adaptive Reconfigurable Controller

In this section, we will present the integrated multiple-model adaptive reconfigurable controller that achieves the objective in the presence of simultaneous loss of effectiveness and freezing or hard-over failures.

In this case, the identification models are of the form

$$\dot{\hat{x}}_{10} = \bar{A}\hat{x}_{20} \tag{55}$$

$$\dot{\hat{x}}_{20} = \Lambda\hat{e}_0 + Ax + B\hat{K}_0 u_c \tag{56}$$

$$\dot{\hat{x}}_{1i} = \bar{A}\hat{x}_{2i} \tag{57}$$

$$\dot{\hat{x}}_{2i} = \Lambda\hat{e}_i + Ax + \bar{B}_i\hat{K}_i u_c + b_i\hat{u}_i, \qquad i = 1, 2, \ldots, m \tag{58}$$

The corresponding controllers are of the form

$$u_{ci} = \hat{K}_i^{-1}W^{-1}B^T(BW^{-1}B^T)^{-1}\Theta_i[\eta - b_i\hat{u}_i]$$

$$i = 0, 1, 2, \ldots, m \tag{59}$$

where $\Theta_0 = I_{k\times k}$, $\hat{u}_0 = 0$, and matrices $\Theta_i$ are given by Eq. (24).

After substituting each $u_{ci}$ from Eq. (59) into the corresponding model (56) and (58) and subtracting Eq. (5), we obtain

$$\dot{\hat{e}}_{mi} = \Lambda_0\hat{e}_{mi}, \qquad i = 0, 1, 2, \ldots, m \tag{60}$$

where $\hat{e}_{mi} = \hat{x}_i - x_m$.

The error model is obtained by subtracting Eqs. (12) and (13) from Eqs. (55–58):

$$\dot{\hat{e}}_{01} = \bar{A}\hat{e}_{02} \tag{61}$$

$$\dot{\hat{e}}_{02} = \Lambda\hat{e}_0 + BU_c\psi_0 \tag{62}$$

$$\dot{\hat{e}}_{i1} = \bar{A}\hat{e}_{i2} \tag{63}$$

$$\dot{\hat{e}}_{i2} = \Lambda_0\hat{e}_i + \bar{B}_i U_c\psi_i, + b_i\phi_i, \qquad i = 1, 2, \ldots, m \tag{64}$$

where $U_c = \text{diag}[u_{c1} \;\; u_{c2} \;\; \ldots \;\; u_{cm}]$ and $\psi_i = [\psi_{1i} \;\; \psi_{2i} \;\; \ldots \;\; \psi_{mi}]^T$ $i = 0, 1, 2, \ldots, m$, and $\psi_{ij} = \hat{k}_{ij} - k_i$, $i = 1, 2, \ldots, m$; and $j = 0, 1, 2, \ldots, m$.

The adaptive laws are of the form

$$\dot{\hat{u}}_i = \underset{[(u_i)_{\min}, (u_i)_{\max}]}{\text{Proj}}\left\{-\gamma_i\hat{e}_i^T P b_i\right\}$$

$$\hat{u}_i(0) \in [(u_i)_{\min}, (u_i)_{\max}], \qquad i = 1, 2, \ldots, m \tag{65}$$

$$\dot{\hat{k}}_{ij} = \underset{[\epsilon_i, 1]}{\text{Proj}}\left\{-\gamma_{ij}\hat{e}_i^T\bar{p}_i u_{ci}\right\}, \qquad \hat{k}_{ij}(0) \in [\epsilon_i, 0]$$

$$i = 1, 2, \ldots, m, \qquad j = 0, 1, 2, \ldots, m \tag{66}$$

The switching scheme is as follows. As in the earlier case, switching among the controllers is based on the performance indices (47). The scheme is started with $u_{co}$ and is implemented by calculating and comparing the preceding indices every $t_s$ instants and finding their minimum. Once the minimum is found, the scheme switches to (or stays at) the corresponding controller.

*Theorem 6:* The preceding switching scheme assures the stability of the system (61–64), (65), and (66), and guarantees that $\lim_{t\to\infty}[x(t) - x_m(t)] = 0$.

*Proof:* The proof is given in Appendix E.

*Comments:*

1) Note that the same approach suggested for the case without the loss of effectiveness carries over to the most complex case as well, and that the proofs related to simpler cases can be effectively used in the proofs of stability in this case.

2) In the case with no loss of effectiveness, the proposed scheme guarantees that the estimate of the failed effector, that is, $\hat{u}_i(t)$, will converge to its true value. However, the scheme does not guarantee that the other estimates will also converge to their true values even if $u_c(t)$ converges to a constant vector. This is because each observer assumes that the no-failure case is the nominal regime, whereas immediately following a failure of the $i$th effector, the new nominal regime is the failure mode of the $i$th effector. Hence, the scheme cannot handle subsequent failures, which can cause significant performance deterioration and even instability of the system. For these reasons, in this paper, we propose a supervisory FDI-adaptive reconfigurable controller (FDI-ARC) scheme. This is discussed in the following section.

## VI. Supervisory FDI-ARC Scheme

The design of the supervisory FDI-ARC scheme is based on the observation that, even when the failure is accurately identified, the remaining estimates do not, in general, converge to their true values. We also note that the described FDI-ARC scheme generates the information about the observer closest to the current plant dynamics, for instance, the $i$th observer, and about the corresponding estimate $\hat{u}_i$. To assure that all of the estimates will converge to their true values, in this paper we propose a supervisory FDI scheme shown in Fig. 4.

The FDI subsystem passes on the information about the observer closest to the current plant dynamics and the corresponding parameter estimate to the parameter estimation subsystem; based on that information and under the assumption that the control input eventually converges to a constant value, the parameter estimation subsystem estimates accurately the remaining parameters. In addition, in the case of failure of the $i$th effector, and based on accurate estimates of all parameters, the decision-making subsystem uses this information online to reset all FDI observers to the new nominal regime, that is, the regime established immediately following the failure.

In this case, the parameter estimation subsystem consists of an additional set of observers that generate estimates $\chi_j$ of $x$. These observers are initiated with $\chi(t_0) = \boldsymbol{x}(t_0)$ and only if there exists an $i > 0$ such that $I_i(t_0) = \min\{I_0(t_0), I_1(t_0), \ldots, I_m(t_0)\}$ (we recall that $i = 0$ corresponds to the no-failure case). The observers are of the form

$$\dot{\hat{\chi}}_{1j} = \bar{A}\hat{\chi}_{2j} \tag{67}$$

$$\dot{\hat{\chi}}_{2j} = \Lambda(\hat{\chi}_j - \boldsymbol{x}) + A\boldsymbol{x} + \bar{B}_j\boldsymbol{u}_c + \boldsymbol{b}_j\theta_j + \boldsymbol{b}_i(\hat{u}_i - u_{ci})$$
$$j = 1, 2, \ldots, m, \quad j \neq i \tag{68}$$

where

$$\dot{\theta}_j = \underset{[(u_j)_{\min}, (u_j)_{\max}]}{\mathrm{Proj}} \left\{-\gamma_j(\hat{\chi}_j - \boldsymbol{x})^T P\boldsymbol{b}_j\right\}$$

$$\theta_j(0) \in [(u_j)_{\min}, (u_j)_{\max}], \qquad j = 1, 2, \ldots, m, \quad j \neq i \tag{69}$$

Note that, in the case of failure of the $i$th effector, we will have

$$\dot{\boldsymbol{x}}_1 = \bar{A}\boldsymbol{x}_2 \tag{70}$$

$$\dot{\boldsymbol{x}}_2 = A\boldsymbol{x} + \bar{B}_i\boldsymbol{u}_c + b_i\bar{u}_i \tag{71}$$

$$\dot{\hat{\chi}}_{1j} = \bar{A}\hat{\chi}_{2j} \tag{72}$$

$$\dot{\hat{\chi}}_{2j} = \Lambda(\hat{\chi}_j - \boldsymbol{x}) + A\boldsymbol{x} + \bar{B}_i\boldsymbol{u}_c + b_i\hat{u}_i + b_j(\theta_j - u_{cj})$$
$$j = 1, 2, \ldots, m, \quad j \neq i \tag{73}$$

Hence, each of the preceding observers (except for the $i$th observer), will contain the estimate of $\bar{u}_i$ (which comes from the FDI subsystem); the $j$th observer also contains its own estimate of $u_j (j \neq i)$. If $\boldsymbol{u}_c(t)$ converges to a constant vector, because $\hat{u}_i(t)$ tends asymptotically to $\bar{u}_i$, we can readily show that all estimates $\theta_j$ will tend asymptotically to their true values. These observations are summarized in the following theorem:

*Theorem 7:* Let $\boldsymbol{\theta} = [\theta_1\ \theta_2\ \ldots\ \theta_{i-1}\ \theta_{i+1}\ \ldots\ \theta_m]^T$. Assume that the $i$th effector has failed and that $\lim_{t\to\infty} \boldsymbol{u}_c(t) = \bar{\boldsymbol{u}}_c$. Then the supervisory FDI-ARC scheme guarantees that 1) $\lim_{t\to\infty} \theta(t) = \bar{\boldsymbol{u}}_c$ and 2) any lock-in-place or hard-over failure that occurs after some time interval $\tau_1$ will be effectively detected and accommodated.

*Proof:* If the $i$th effector has failed, the FDI subsystem described in the preceding section assures that $\lim_{t\to\infty} \phi_i(t) = 0$ and that the scheme switches to $\boldsymbol{u}_{ci}$. This information activates the parameter estimation subsystem. Let $\epsilon_{1j} = \hat{\chi}_{1j} - \boldsymbol{x}_1$, $\epsilon_{2j} = \hat{\chi}_{2j} - \boldsymbol{x}_2$, and $\epsilon_j = [\epsilon_{1j}^T\ \ \epsilon_{2j}^T]^T$. Using Eqs. (70–73), we obtain the following error model:

$$\dot{\epsilon}_{1j} = \bar{A}\epsilon_{2j} \tag{74}$$

$$\dot{\epsilon}_{2j} = \Lambda\epsilon_j + \boldsymbol{b}_i\phi_i + \boldsymbol{b}_j\psi_j, \qquad j = 1, 2, \ldots, m, \quad j \neq i \tag{75}$$

where $\psi_j = \theta_j - u_{cj}$. Then $\dot{\psi}_j = \dot{\theta}_j - \dot{u}_{cj}$, where $\dot{\theta}_j$ is given in Eq. (69).

We next choose

$$V_j(\epsilon_j, \psi_j) = \tfrac{1}{2}\left(\epsilon_j^T P\epsilon_j + \psi_j^2/\bar{\gamma}_j\right) \tag{76}$$

where $\bar{\gamma}_j > 0$. Its first derivative along the motion of the $j$th observer yields

$$\dot{V}_j(\epsilon_j, \psi_j) \leq -\tfrac{1}{2}\lambda_m\|\epsilon_j\|^2 + \epsilon_j^T P\boldsymbol{b}_i\phi_i - \psi_j\dot{u}_{cj}/\bar{\gamma}_j \tag{77}$$

where the properties of adaptive algorithms with projection were used. Based on the results of earlier theorems, it can be readily shown that $\dot{u}_{cj}$ is bounded. Because $\phi_i$ is bounded, and adaptive algorithms with projection are used to adjust $\phi_j$, there exist $c_1 > 0$ and $c_2 > 0$ such that

$$\dot{V}_j(\epsilon_j, \psi_j) \leq -\tfrac{1}{2}\lambda_m\|\epsilon_j\|^2 + c_1\|\epsilon_j\| + c_2 \tag{78}$$

Using the arguments from Ref. 14, we can readily demonstrate that there exists a $c > 0$ such that $\|\epsilon_j(t)\| \leq c$ for all $t \geq t_0$. Hence, $V_j(t)$ is also bounded.

We further integrate $\dot{V}_j$ from Eq. (48) to obtain

$$V_j(0) - V_j(\infty) \geq \frac{1}{2}\lambda_m \int_0^\infty \|\hat{\boldsymbol{e}}_i(t)\|^2\,\mathrm{d}t$$
$$- c_1' \int_0^\infty |\phi_i(t)|\,\mathrm{d}t - c_2' \int_0^\infty |\dot{u}_{cj}(t)|\,\mathrm{d}t \tag{79}$$

for some constants $c_1' >$ and $c_2' > 0$. Because $\phi_i(t)$ tends to zero, and $\boldsymbol{u}_c(t)$ tends to $\bar{\boldsymbol{u}}_c$, the last two terms on the right-hand side of the

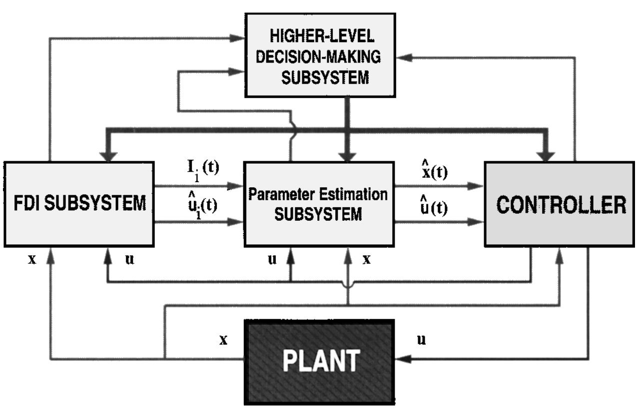


**Fig. 4 Structure of the proposed supervisory FDI-ARC system (© 1999–2002 Scientific Systems Co., Inc.).**

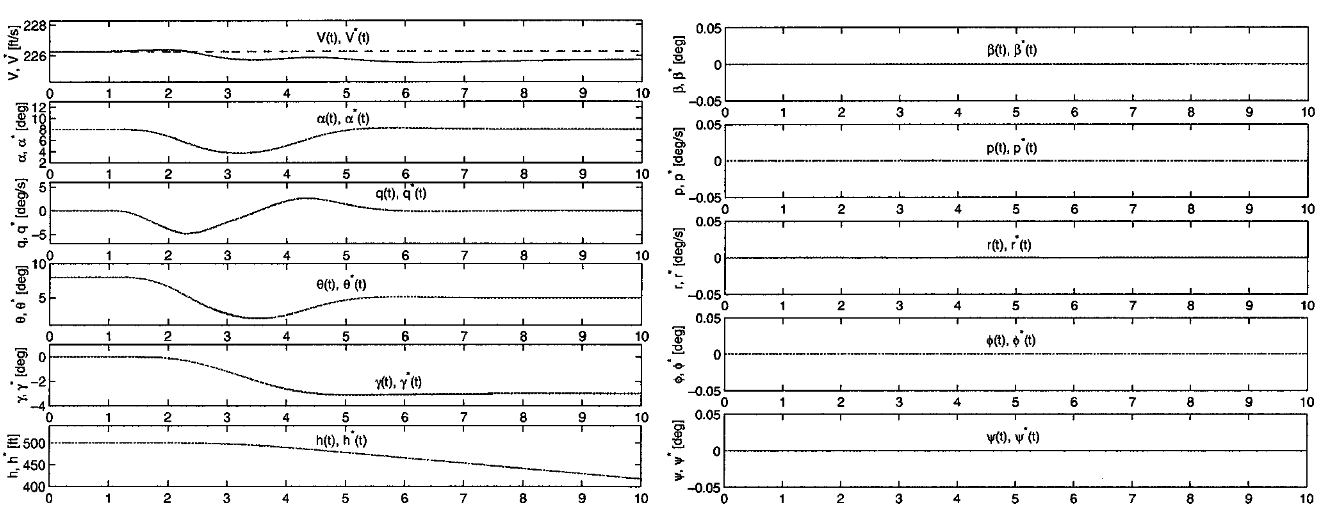


**Fig. 5 State response of the F/A-18C and D aircraft with the baseline controller: case with no failures or disturbances.**

preceding expression are bounded. Hence, $\epsilon_j \in \mathcal{L}^2$. From Eqs. (74) and (75), $\dot{\epsilon}_j$ is bounded. Hence, $\lim_{t\to\infty} \epsilon_j(t) = 0$. The adaptive system (74), (75), and (69) contains only one adjustable parameter $\psi_j$, that is, $\theta_j$, whereas $\epsilon_j$ tends to zero asymptotically. Because $\epsilon_j = 0$ and $\psi_j = 0$ is a unique equilibrium state, it follows that $\lim_{t\to\infty} \psi_j(t) = 0$. Because the same holds for all $j \neq i$, Theorem 7, part 1 holds.

We next define a vector $\boldsymbol{\omega}$ that depends on $\hat{\boldsymbol{e}}_i$, $\boldsymbol{u}_c$, $\epsilon_j$, and $\theta_j$, $j = 1, 2, \ldots, m$, $j \neq i$, that is, $\boldsymbol{\omega} = [\hat{\boldsymbol{e}}_i^T \; \epsilon_1^T \; \epsilon_2^T \; \ldots \; \epsilon_{i-1}^T \; \epsilon_{i+1}^T \; \ldots \; \epsilon_m^T \; \psi_1 \; \psi_2 \; \ldots \; \psi_{i-1} \; \psi_{i+1} \; \ldots \; \psi_m]^T$. This stability analysis, along with that from earlier theorems, implies that all signals in the system will converge to zero in a uniform fashion, including $\boldsymbol{\omega}(t)$. Then there exists a time instant $\tau_1$ such that $\|\boldsymbol{\omega}(t)\| \leq \delta$ for all $t \geq \tau_1$. By choosing $\delta$ properly, the decision-making subsystem resets all observers to the new operating regime. Hence, the initial nominal observer is removed, and the observer corresponding to the failure of the $i$th effector becomes the new nominal observer. All other observers are modified accordingly. We can conclude that this enables the scheme to detect, identify, and accommodate subsequent failures that occur for $t \geq \tau_1$, which completes the proof. □

One of the main features of the proposed scheme is that, because adaptive algorithms with projection are used, we can choose large values of adaptive gains to guarantee fast convergence of $\phi_i(t)$ and $\psi_j(t)$ to zero. We next evaluated the performance obtained using the supervisory FDI-ARC scheme through simulations. This is discussed in the following section.

## VII. Simulation Results

All simulation results are evaluated on a combined linear and nonlinear simulation of F/A-18C/D. The simulation combines the nonlinear aircraft kinematics with linearized control derivatives obtained from a linearized model provided by The Boeing Co. The simulation also includes linear actuator dynamics (second order), as well as engine dynamics (sixth order), along with position and rate limits on the control effectors.

### A. Selected Flight Regime

The simulated flight regime is power approach. Landing is initiated at $t = 0$ from 500 ft (152.4 m) and with forward velocity $V = 228.5$ ft/s (69.65 m/s). The desired response is specified using a suitably defined reference model.

### B. Control Variables

The following control effectors are available for controlling the aircraft: $u_1$ and $u_2$ = left and right leading-edge flap, 2) $u_3$ and $u_4$ = left and right stabilator, 3) $u_5$ and $u_6$ = left and right rudder toe-in, 4) $u_7$ = engine power lever, and 5) $u_8$ and $u_9$ = left and right aileron.

### C. Control Objective

The objective is to achieve fast and accurate capture of the glide slope of 3 deg. The desired speed of response is specified by the requirement that the flight-path angle achieves a value of −2 deg in 3.5–3.8 s.

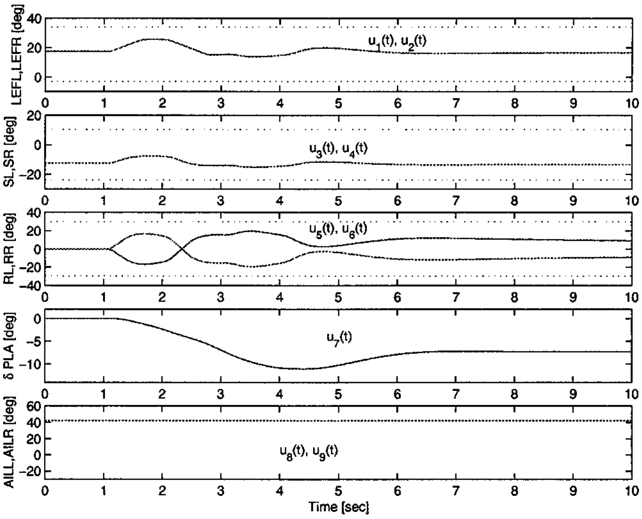


**Fig. 6 Input response of the F/A-18C and D aircraft with the baseline controller: case with no failures or disturbances.**

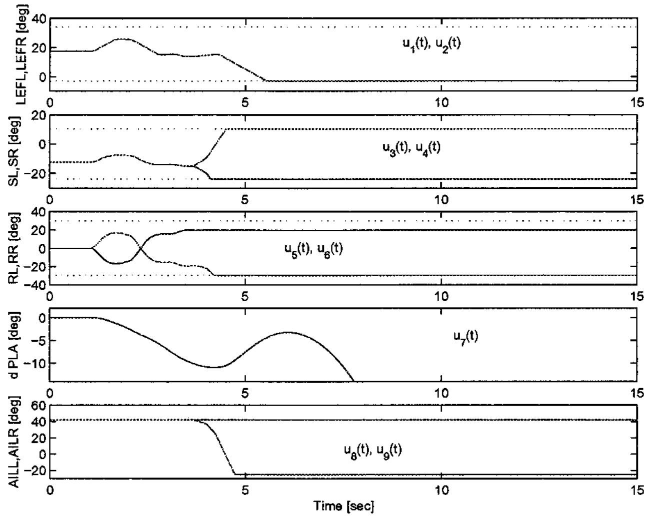


**Fig. 8 Input response of the F/A-18C and D aircraft with the baseline controller: case of left rudder lock-in-place and the roll-axis disturbance.**

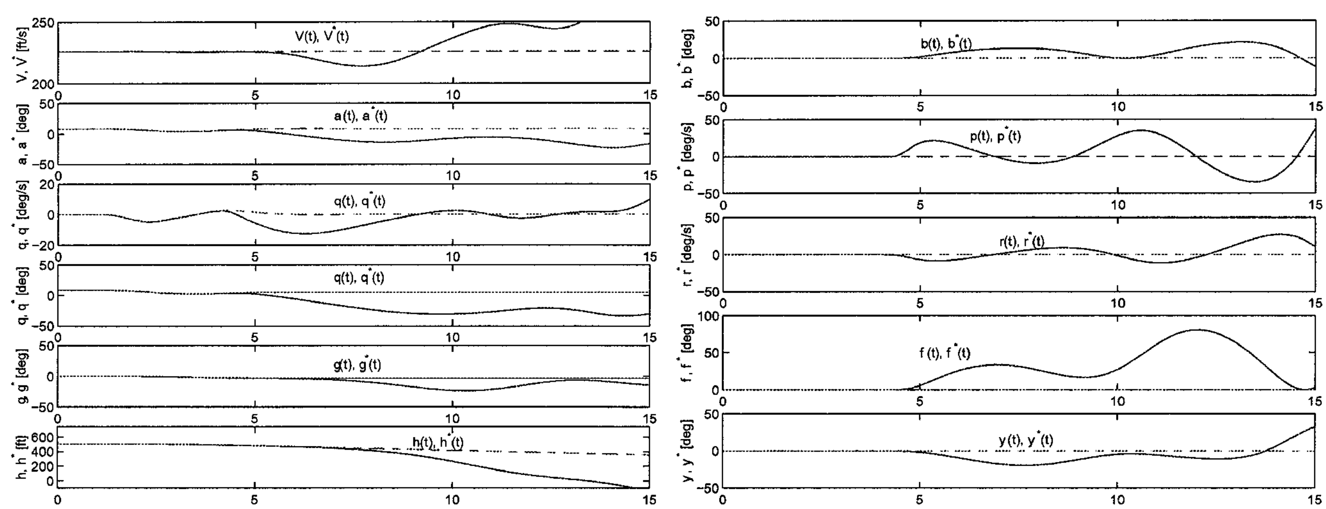


**Fig. 7 State response of the F/A-18C and D aircraft with the baseline controller: case of left rudder lock-in-place and the roll-axis disturbance.**

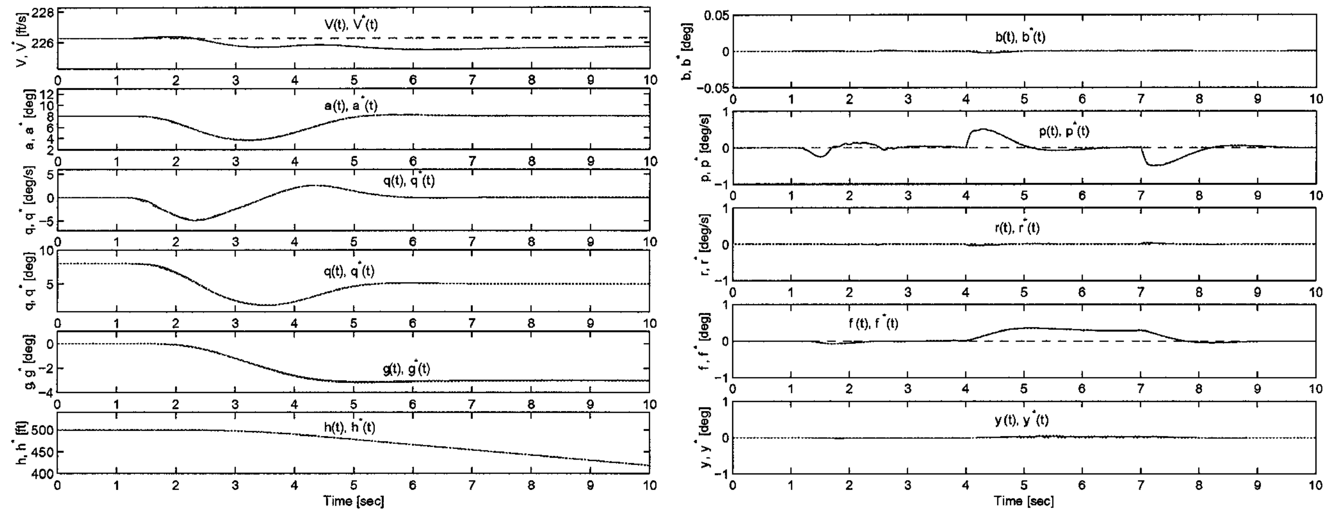


**Fig. 9 State response of the F/A-18C and D aircraft with the adaptive reconfigurable controller: case of 50% left stabilator damage, left rudder lock-in-place, and the roll-axis disturbance.**

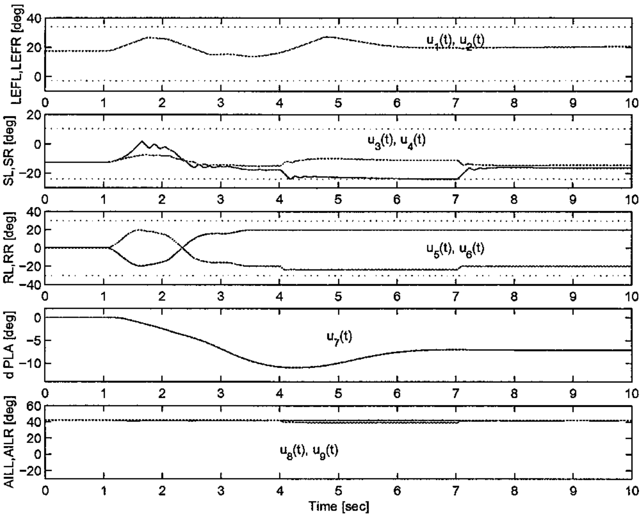


**Fig. 10 Input response of the F/A-18C and D aircraft with the adaptive reconfigurable controller: case of 50% left stabilator damage, left rudder lock-in-place, and the roll-axis disturbance.**

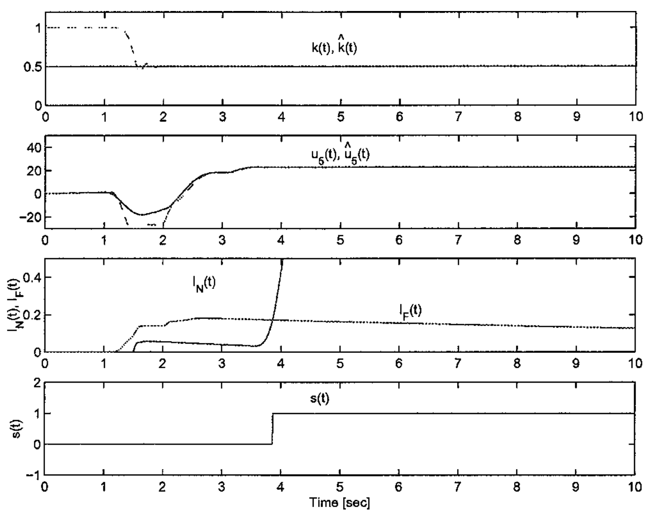


**Fig. 11 Failure-related parameter estimates, performance indices, and the switching sequence in the case of the combined failure.**

### D. Failure Scenario

We have chosen the following failure scenario: Left stabilator undergoes 50% damage at $t = 0$ s, left rudder locks in place at $t = 3.5$ s, and there is roll axis disturbance at $t = 4$ s.

### E. Baseline Controller

We first simulated the aircraft with the baseline controller in the ideal case, that is, the case without failures or disturbances. The resulting response is shown in Figs. 5 and 6. The control allocation is chosen to minimize the sensitivity to failures. Our next simulation was carried out in the case when the left rudder locks in pace at $t = 3.5$ s. The response from Figs. 7 and 8 shows that the closed-loop system becomes unstable. This is due to saturation of most of the control effectors. Note that the system becomes unstable even in the case when there are no disturbances acting on the aircraft. Hence, the baseline controller cannot handle critical failures, and control reconfiguration is needed to assure the stability of the closed-loop system.

### F. ARC

As shown in Ref. 15, our adaptive reconfigurable controller assures excellent response in the following cases: 1) 50% left stabilator damage at $t = 0$ s and disturbance at $t = 4$ s, 2) left rudder failure at $t = 3.5$ s and disturbance at $t = 4$ s, and 3) combined failure, 50% left stabilator damage during straight flight ($t = 0$), and left rudder lock in place at $t = 3.5$ s. The response in the latter case is shown in Figs. 9 and 10. Note that the resulting response is excellent despite the presence of multiple failures and disturbance. The estimates of $k(t)$ and $u_5(t)$, as well as the corresponding performance indices and the switching sequence, are shown in Fig. 11. It is seen that the scheme immediately switches to the right observer and controller based on the relative value of the performance indices and that the estimates of the failure-related parameters quickly converge to their true values. The overall scheme is highly robust to the disturbance in the roll axis.

## VIII. Conclusions

In this paper we propose a new parameterization for the modeling of control effector failures in flight control applications. The failures include lock in place, hard over, and loss of effectiveness. It is shown that the resulting representation leads naturally to a multiple-model formulation of the corresponding control problem that can be solved using a multiple-model adaptive reconfigurable control approach. In this context, stable multiple-model adaptive reconfigurable control algorithms were derived for several cases of increasing complexity, including the most complex case, when one of the effectors undergoes lock-in-place or hard-over failure, and all others loose effectiveness. In all cases, the stability of the overall reconfigurable control system was demonstrated using multiple Lyapunov functions, extensions of the Lyapunov method, and the separation between identification and control arising in the context of indirect adaptive control.

We emphasize that the stability results from the paper are derived in the ideal case, that is, in the case with no disturbances, control input saturation, and actuator dynamics. From that point of view, the results from the paper are suitable for initial analysis and as a starting point in the procedure in which magnitudes of different disturbances, control input saturation, effect of nonlinear coupling, and actuator dynamics need to be taken into account during the verification and validation of the control algorithm. Because of the properties of adaptive algorithms with projection, we can readily demonstrate that the system will be robust in the presence of parametric uncertainty, time-varying parameters, bounded external disturbances, noise, and some classes of unmodeled dynamics. A detailed analysis of the robustness and performance of the overall system in the presence of such perturbations is a subject of our current investigation.

Another important question that arises in the context of flight control is that of both position and rate saturation of control effectors. Because the case of adaptive algorithms with projection transient bounds on the output error can be readily calculated, we are currently working on relating these bounds to the state sets obtained from the input saturation bounds.

We also plan to extend this approach to the case of uncertain system parameters, as well as to nonlinear aircraft models.

## Appendix A: Proof of Theorem 1

*Proof:* We first note that Eq. (32) implies that $\lim_{t\to\infty}\hat{e}_0(t)=0$, whereas Eq. (33) implies that $\lim_{t\to\infty}\hat{e}_{m0}(t)=0$. These two conditions also imply that $\lim_{t\to\infty}[x(t)-x_m(t)]=0$, which proves part 2 of Theorem 1. To prove part 1, we now need to show that all errors $\hat{e}_i$, $i=1,2,\ldots,m$ are bounded.

Let the Lyapunov function candidate for the $i$th identification model be of the form

$$V_i(\hat{e}_i,\phi_i)=\tfrac{1}{2}\left(\hat{e}_i^T P\hat{e}_i+\phi_i^2\big/\gamma_i\right)$$

Because $\dot{\phi}_i=\dot{\hat{u}}_i-\dot{u}_i$, using the properties of adaptive algorithms with projection (see Appendix I) and Eq. (36), its first derivative along the motions of Eqs. (28), (29), and (37) yields

$$\dot{V}_i(\hat{e}_i,\phi_i)\le-\tfrac{1}{2}\hat{e}_i^T Q\hat{e}_i-\phi_i\dot{u}_i$$

We now note that, because there is no failure, $u_i=u_{ci}$. Also, each $u_{ci}$ is bounded because $x$, $x_m$, $r$, and $\hat{u}_i$ are bounded and

$$\dot{u}_{ci}=W^{-1}B^T(BW^{-1}B^T)^{-1}\Theta_i[\dot{\eta}-b_i\dot{\hat{u}}_i]$$

where $\dot{\eta}=-(A-\Lambda)\dot{x}-(\Lambda-A_m)\dot{x}_m+B_m\dot{r}$. Because $\dot{x}=Ax+Bu_c$, and because $u_c$ is bounded, it also follows that $\dot{x}$ is bounded. Because $r$ is bounded, $x_m$ is also bounded, which implies that $\dot{x}_m$ is bounded as well. Because $\dot{r}\in\mathcal{L}^\infty$, it follows that $\dot{\eta}(t)$ is also bounded. In addition, from the adaptive algorithms (37), $\dot{\hat{u}}_i(t)$ is bounded. The preceding analysis implies that there exists a constant $c_{ui}>0$ such that

$$\|\dot{u}_{ci}(t)\|\le c_{ui},\qquad \forall t\ge t_0$$

Because the adaptive algorithms (37) result in $|\phi_i(t)|\le(u_i)_{\max}-(u_i)_{\min}$, we also have

$$\dot{V}_i(\hat{e}_i,\phi_i)\le-\tfrac{1}{2}\lambda_m\|\hat{e}_i\|^2+c_i$$

where $\lambda_m$ denotes the minimum eigenvalue of $Q$ and $c_i=[(u_i)_{\max}-(u_i)_{\min}]c_{ui}$. It follows from Ref. 14 (see Appendix H) that $\hat{e}_i$ is bounded. Because this holds for any $i=1,2,\ldots,m$, this completes the proof. □

## Appendix B: Proof of Theorem 2

*Proof:* Let the Lyapunov function candidate for the jth identification model be of the form

$$V_j(\hat{e}_j,\phi_j)=\tfrac{1}{2}\left(\hat{e}_j^T P\hat{e}_j+\phi_j^2\big/\gamma_j\right)$$

Because in the case of lock-in-place or hard-over failure of the $j$th effector we have $\dot{u}_{cj}(t)=0$ for all $t\ge t_{Fj}$, it follows that $\dot{\phi}_j(t)=\dot{\hat{u}}_j(t)$ over the same interval. By the use of the properties of adaptive algorithms with projection, the first derivative of $V_j$ along the motions of Eqs. (42), (43), and (37) for $i=j$ yields

$$\dot{V}_j(\hat{e}_j,\phi_j)\le-\tfrac{1}{2}\hat{e}_j^T Q\hat{e}_j\le 0$$

It follows that $\hat{e}_j\in\mathcal{L}^2\cap\mathcal{L}^\infty$ (see Appendix G). However, this does not imply that $\lim_{t\to\infty}\hat{e}_j(t)=0$. Because Eq. (44) implies that $\lim_{t\to\infty}\hat{e}_{mj}(t)=0$, and because $x_m$ is bounded, it follows that $\hat{x}_j$ is bounded. Because $\hat{e}_j$ is bounded, this in turn implies that $x$ is bounded as well. Because $x$, $x_m$, $r$, and $\hat{u}_i$ are bounded, it follows that each $u_{ci}$ is bounded, including $u_{cj}$. This implies that $\dot{\hat{e}}_j$ is bounded, which implies (see Appendix G), that $\lim_{t\to\infty}\hat{e}_j(t)=0$. Because both $\hat{e}_{mj}(t)$ and $\hat{e}_j(t)$ tend to zero asymptotically, it follows that $\lim_{t\to\infty}[x(t)-x_m(t)]=0$, which proves part 2 of Theorem 2.

We now need to prove that the errors $\hat{e}_0$ and $\hat{e}_i$, $i=1,2,\ldots,m$, $i\ne j$, are bounded. Let the Lyapunov function candidate for the identification model (40) and (41) be of the form

$$V_0(\hat{e}_0)=\tfrac{1}{2}\hat{e}_0^T P\hat{e}_0$$

Its first derivative along the motions of Eqs. (40) and (41) yields:

$$\dot{V}_0(\hat{e}_0)=-\tfrac{1}{2}\hat{e}_0^T Q\hat{e}_0+\hat{e}_0^T Pb_j(u_{cj}-\bar{u}_{cj})$$

Because each $u_{ci}$ is bounded, then there exists a constant $c_{uj}$ such that $|u_{cj}(t)|\le c_{uj}$ for all $t\ge t_{Fj}$. Hence,

$$\dot{V}_0(\hat{e}_0)\le-\tfrac{1}{2}\lambda_m\|\hat{e}_0\|^2+c_0\|\hat{e}_0\|$$

where $c_0=\|Pb_j\|(c_{uj}+\bar{u}_{cj})$. It again follows from Ref. 14 (see Appendix H) that $\hat{e}_0$ is bounded.

Along exactly the same lines as in the proof of Theorem 1 it follows that all errors $\hat{e}_i$, $i=1,2,\ldots,m$, $i\ne j$ are also bounded, which proves part 1 of Theorem 2.

In addition, because the system (42) and (43) is linear, its unique equilibrium state is $(\hat{e}_j=0,\ \phi_j=0)$. Because $\lim_{t\to\infty}\hat{e}_j(t)=0$, it follows that $\lim_{t\to\infty}\phi_j(t)=0$, which completes the proof. □

## Appendix C: Proof of Theorem 4

*Proof:* Because the design of the identification models is under the discretion of the designer and because $x(t)$ is measurable, all models can be initiated with $\hat{x}_i(0)=x(0)$ at $t_0=0$. Because the system is started with $u_{co}$, this results in $\hat{e}_0(t)=0$, for all $t\in\mathcal{S}_t=\{t:0\le t<t_{Fj}\}$. Because all other models have forcing terms due to the terms $\dot{u}_i(t)$, the indices $I_j(t)$, $j=1,2,\ldots,m$, will be greater than zero, so that $I_0(t)$ will be minimum over $\mathcal{S}_t$, and the scheme will stay at $u_{co}$. If there is a failure of the $j$th effector, the $j$th identification model will correspond to the current plant dynamics. Using this model, we can, hence, readily show that $\hat{e}_j\in\mathcal{L}^2\cap\mathcal{L}^\infty$. From Theorems 1 and 2 we know that the adaptive algorithms (37) assure that all identification errors $\hat{e}_i$, $i=0,1,2,\ldots,m$, will be bounded regardless of the current controller. Furthermore, it is clear that, with any current controller, for example, $u_{ci}$, the corresponding identification model assures that $\hat{e}_{mi}$ is bounded, which, because $x_m$ is bounded, implies that $\hat{x}_i$ is bounded as well. Because $x=-\hat{e}_i+\hat{x}_i$, it follows that $x$ is bounded as well. The latter also implies that every $u_{ci}$ is bounded, from where it follows that $\dot{\hat{e}}_j$ is also bounded. Hence (see Appendix G), $\lim_{t\to\infty}\hat{e}_j(t)=0$. This implies that, due to the term $\exp[-\lambda(t-\tau)]$, $\lim_{t\to\infty}I_j(t)=0$. Because all other models will have forcing terms, and because all indices have integral terms so that their errors will accumulate over a time interval, it follows that there exists a time instant $t_j>t_{Fj}$ such that $I_j(t)=\min_i\{I_i(t)\}$ for $t\ge t_j$. Hence, the scheme will switch to $u_{cj}$. Based on the results of Theorems 2 and 3, we can now conclude that all signals are bounded, and that $\lim_{t\to\infty}\hat{e}_j(t)=\lim_{t\to\infty}\hat{e}_{mj}(t)=\lim_{t\to\infty}[x(t)-x_m(t)]=0$. □

## Appendix D: Proof of Theorem 5

*Proof:* Let $\Phi_k = \hat{K} - K = \mathrm{diag}[\phi_{k1}\ \phi_{k2}\ \dots\ \phi_{km}]$. From Eqs. (12), (13), (48), and (49) we obtain the following error model:

$$\dot{\hat{e}}_{01} = \bar{A}\hat{e}_{02}, \qquad \dot{\hat{e}}_{02} = \Lambda\hat{e}_0 + B\Phi_k u_c$$

Because $\Phi_k$ is a diagonal matrix, this error model can be rewritten as

$$\dot{\hat{e}}_{01} = \bar{A}\hat{e}_{02}, \qquad \dot{\hat{e}}_{02} = \Lambda\hat{e}_0 + BU_c\phi_k$$

where $U_c = \mathrm{diag}[u_{c1}\ u_{c2}\ \dots\ u_{cm}]$, and $\phi_k = [\phi_{k1}\ \phi_{k2}\ \dots\ \phi_{km}]T$.

We now choose

$$V(\hat{e}_0, \phi_k) = \tfrac{1}{2}\left[\hat{e}_0^T P\hat{e}_0 + \phi_k^T \Gamma_k^{-1}\phi_k\right]$$

where $\Gamma_k = \mathrm{diag}[\gamma_{k1}\ \gamma_{k2}\ \dots\ \gamma_{km}]$. It follows that

$$\dot{V}(\hat{e}_0, \phi_k) = -\tfrac{1}{2}\hat{e}_0^T Q\hat{e}_0 + \hat{e}_0^T \bar{P}U_c\phi_k + \phi_k^T\Gamma_k^{-1}\dot{\phi}_k$$

Based on Proposition 1,

$$\dot{V}(\hat{e}_0, \phi_k) \le -\tfrac{1}{2}\lambda_m\|\hat{e}_0\|^2 - \phi_k^T\Gamma_k^{-1}\dot{k}$$

Because $\phi_k$ is bounded by the choice of adaptive laws, from assumption 2, there exists a constant $c_k > 0$ such that $|\phi_k^T(t)\Gamma_k^{-1}\dot{k}(t)| \le c_k$ for all time. Hence,

$$\dot{V}(\hat{e}_0, \phi_k) \le -\tfrac{1}{2}\lambda_m\|\hat{e}_0\|^2 + c_k$$

which implies (see Appendix H) that $\hat{e}_0 \in \mathcal{L}^\infty$. Hence, $V$ is also bounded. We further integrate the expression (F1) to obtain

$$V(0) - V(\infty) \ge \frac{1}{2}\lambda_m\int_0^\infty \|\hat{e}_0(t)\|^2\,\mathrm{d}t + \int_0^\infty \phi_k^T(t)\Gamma_k^{-1}\dot{k}(t)\,\mathrm{d}t$$

Because $\phi_k$ is bounded and $\dot{k} \in \mathcal{L}^2$, it follows that $\hat{e}_0 \in \mathcal{L}^2$. However, this does not imply that either $\hat{e}_0$ or $x$ are bounded, and further analysis is needed.

We next substitute Eq. (50) into Eq. (49) and subtract Eq. (5) to obtain

$$\dot{\hat{e}}_{m0} = \Lambda_0\hat{e}_{m0}$$

where $\hat{e}_{m0} = \hat{x}_0 - x_m$. Hence, $\hat{e}_{m0}$ is bounded and $\lim_{t\to\infty}\hat{e}_m(t) = 0$. Because $x_m$ is bounded by the choice of the reference model, we can conclude that $x$ is bounded as well. This in turn implies that $u_c$ is also bounded.

These properties imply that $\dot{\hat{e}}_0$ is bounded, and, because $\hat{e}_0 \in \mathcal{L}^2 \cap \mathcal{L}^\infty$, it follows that (see Appendix G) $\lim_{t\to\infty}\hat{e}_0(t) = 0$. Because $\lim_{t\to\infty}\hat{e}_m(t) = 0$, the latter condition also implies that $\lim_{t\to\infty}[x(t) - x_m(t)] = 0$. □

## Appendix E: Proof of Theorem 6

*Proof*: The scheme is started with $\hat{x}_i(0) = x(0)$, $i = 0, 1, 2, \dots, m$ and with $u_{co}$. Even though the scheme is started with $u_{co}$, it might not stay at $u_{co}$ if there is a sudden variation of $K(t)$ because in the transient, some other model may be closer to the new operating regime. However, even in such a case the system is guaranteed to switch back eventually to $u_{co}$ as shown hereafter.

If there are no other failures except for loss of effectiveness, using the results of Theorem 5, we can conclude that $\hat{e}_0$, $\hat{e}_{m0}$, and $x$ are bounded. To show that in this case all other signals are also bounded, we first choose

$$V_i(\hat{e}_i, \phi_i, \psi_i) = \tfrac{1}{2}\left[\hat{e}_i^T P\hat{e}_i + \phi_i^2/\gamma_i + \psi_i^T\Gamma_{ki}^{-1}\psi_{ki}\right]$$

where $\Gamma_{ki} = \mathrm{diag}[\gamma_{1i}\ \gamma_{2i}\ \dots\ \gamma_{mi}]$, $i = 1, 2, \dots, m$. Its first derivative along the motion of the $i$th model yields

$$\dot{V}_i(\hat{e}_i, \phi_i, \psi_i) \le -\tfrac{1}{2}\lambda_m\|\hat{e}_i\|^2 - \phi_i\dot{u}_i/\gamma_i - \psi_i^T\Gamma_{ki}^{-1}\dot{k}$$

where the properties of adaptive algorithms with projections were used (Appendix I), along with Proposition 1. Similarly, as in the proofs of Theorems 1 and 5, we can show that signals $\dot{u}_i$ are bounded along with the errors $\hat{e}_i$. From Theorem 5, it follows that in the presence of the loss of effectiveness, all signals are bounded and $\hat{e}_0(t)$, $\hat{e}_{m0}(t)$, and $[x(t) - x_m(t)]$ tend to zero asymptotically. Because all other models have forcing terms and indices $I_i(t)$ have integral terms, the errors will accumulate over a time interval and the indices $I_i(t)$, $i = 1, 2, \dots, m$, will be greater than zero. Because $\hat{e}_0(t)$ tends to zero and because all indices have the exponentially decaying term $\exp[-\lambda(t-\tau)]$, it follows that $\lim_{t\to\infty} I_0(t) = 0$. Hence, even if in the transient the system switched to a different controller, there exists a time instant $t_0$ such that it will switch back to $u_{co}$ for $t \ge t_0$.

In the case of simultaneous loss of effectiveness and lock in place or hard over of the $j$th effector, we have that

$$\dot{\hat{e}}_{10} = \bar{A}\hat{e}_{20}, \qquad \dot{\hat{e}}_{20} = \Lambda_0\hat{e}_0 + \bar{B}_j U_c\psi_0 + b_j(u_{cj} - \bar{u}_{cj})$$

$$\dot{\hat{e}}_{1j} = \bar{A}\hat{e}_{2j}, \qquad \dot{\hat{e}}_{2j} = \Lambda_0\hat{e}_j + \bar{B}_j U_c\psi_j + b_j\phi_j, \qquad \dot{\hat{e}}_{1i} = \bar{A}\hat{e}_{2i}$$

$$\dot{\hat{e}}_{2i} = \Lambda\hat{e}_i + \bar{B}_j U_c\psi_i + b_i\phi_i, \qquad i = 1, 2, \dots, m, \quad i \ne j$$

where $\dot{u}_j(t) = 0$ and $\bar{u}_{cj} = u_{cj}(t_{Fj})$. Because of the former condition, from the expression (80) for $i = j$ and assumption 2, we can conclude that $\hat{e}_j \in \mathcal{L}^2 \cap \mathcal{L}^\infty$. With any controller $u_{ci}$, the corresponding identification model assures that $\hat{e}_{mi}$ is bounded, which, based on the boundedness of $x_m$, implies that $\hat{x}_j$, and, consequently, $x$ are bounded as well. This implies that every $u_{ci}$ is bounded, from where it follows that $\dot{\hat{e}}_j$ is also bounded. Hence (see Appendix H), $\lim_{t\to\infty}\hat{e}_j(t) = 0$. This implies that, due to the term $\exp[-\lambda(t-\tau)]$, $\lim_{t\to\infty} I_j(t) = 0$. Because all other models will have forcing terms, and all indices have integral terms so that their errors will accumulate over a time interval, it follows that there exists a time instant $t_j > t_{Fj}$ such that $I_j(t) = \min_i\{I_i(t)\}$ for $t \ge t_j$. Hence, the scheme will switch to $u_{cj}$. Based on the results of Theorems 2 and 3, we can now conclude that all signals are bounded and that $\lim_{t\to\infty}\hat{e}_j(t) = \lim_{t\to\infty}\hat{e}_{mj}(t) = \lim_{t\to\infty}[x(t) - x_m(t)] = 0$. □

## Appendix F: Pseudoinverse Control Algorithm Based on Ref. 16

The control design problem in the case when the number of available control inputs exceeds the number of outputs can be efficiently studied within the framework of constrained optimization. The objective is to minimize the total control effort, which can be formally stated as

$$\min_u J = \min_u \tfrac{1}{2}u^T W u$$

where $W = W^T > 0$. We commonly choose $W = \mathrm{diag}[w_1\ w_2\ \dots\ w_m]^T$. This criterion is subject to the constraint $\dot{e} = A_m e$ (where $e = x - x_m$, $A_m$ is asymptotically stable, and $x_m$ is the output of a reference model $\dot{x}_m = A_m x_m + B_m r$), for specified $x$ and $x_m$. Such a constraint represents the desired behavior of the closed-loop system and is well suited for the tracking problem considered in this paper. In Ref. 16, the preceding criterion was subject to $\dot{x} = Ax + Bu$. We next consider the following.

*Theorem F1:* Control law

$$u = W^{-1}B^T(BW^{-1}B^T)^{-1}\{-Ax + A_m x + B_m r\} \qquad \text{(F1)}$$

minimizes the preceding criterion.

*Proof:* By the use of the Lagrange multiplier formulation and $\dot{e} - A_m e = Ax + Bu - A_m x - B_m r$, the problem reduces to

$$\min_u \left\{\tfrac{1}{2}u^T W u - \lambda^T(Ax + Bu - A_m x - B_m r)\right\}$$

where $\lambda$ denotes the Lagrange multiplier. Partial derivative with respect to $u$ yields $u^T W + \lambda^T B = 0$, or

$$u = W^{-1}B^T\lambda$$

Substituting this expression into $Ax + Bu - A_m x - B_m r$ and solving for $\lambda$ now yields Eq. (F1), which completes the proof. □

## Appendix G: Stability Analysis of Adaptive Systems[13]

*Theorem G1:* Let an error model be of the form

$$\dot{\boldsymbol{e}} = \Lambda_0 \boldsymbol{e} + B_0 \phi^T \omega(t, \boldsymbol{e}), \qquad \dot{\phi} = -\omega(t, \boldsymbol{e}) B_0^T P \boldsymbol{e} \tag{G1}$$

where $\Lambda_0$ is asymptotically stable, $B_0$ is known, $P$ is given by Eq. (36), and $\omega(t, \boldsymbol{e})$ is bounded for bounded $\boldsymbol{e}$ and for all time. Then $\lim_{t\to\infty} \boldsymbol{e}(t) = 0$.

*Proof:* Let the Lyapunov function candidate be of the form

$$V(\boldsymbol{e}, \phi) = \tfrac{1}{2}[\boldsymbol{e}^T P \boldsymbol{e} + \phi^T \phi]$$

First derivative of $V$ along the motions of Eq. (G1) yields

$$\dot{V}(\boldsymbol{e}, \phi) = -\tfrac{1}{2}\boldsymbol{e}^T Q \boldsymbol{e} \leq -\tfrac{1}{2}\lambda_m \|\boldsymbol{e}\|^2 \leq 0 \tag{G2}$$

where $\lambda_m$ denotes the minimum eigenvalue of $Q$. It follows that the equilibrium $(\boldsymbol{e} = 0, \phi = 0)$ of Eq. (G1) is stable in the sense of Lyapunov. This implies that $\boldsymbol{e}$ and $\phi$ are bounded, that is, $\boldsymbol{e} \in \mathcal{L}^\infty$ and $\phi \in \mathcal{L}^\infty$. Hence, $V$ is bounded as well. On integrating Eq. (G2), we obtain

$$V(0) - V(\infty) \geq \frac{1}{2}\lambda_m \int_{t_0}^{\infty} \|e(t)\|^2 \,\mathrm{d}t$$

Because the term on the left-hand side of this expression is bounded, it follows that $\boldsymbol{e} \in \mathcal{L}^2$. From Eq. (G1), we also know that $\dot{\boldsymbol{e}}$ is bounded, so that, from the Barbalaat's lemma (see, for example Ref. 13), it follows that $\lim_{t\to\infty} \boldsymbol{e}(t) = 0$. □

## Appendix H: Extensions of the Lyapunov Method from Ref. 14

*Theorem H1:* Let $(\xi_1 = 0, \xi_2 = 0)$ be an equilibrium of the system $\dot{\xi}_i = f_i(\xi)$, $i = 1, 2$, where $\xi = |\xi_1^T\, \xi_2^T|$. If $\xi_2 \in \mathcal{L}^\infty$, and if there exists a tentative Lyapunov function $V(\xi)$ for the system such that

$$\dot{V}(\xi) \leq -c_{\xi 0}\|\xi_1\|^2 + c_{\xi 1}\|\xi_1\| + c_{\xi 2}$$

then $\xi_1 \in \mathcal{L}^\infty$.

*Proof:* We first rewrite this expression in the form

$$\dot{V}(\xi) \leq -c_{\xi 0}\left[\|\xi_1\|^2 - \bar{c}_1\|\xi_1\| - \bar{c}_2\right]$$

where $\bar{c}_i = c_{\xi i}/c_{\xi 0}$, $i = 1, 2$. From the expression, it follows that $\dot{V} > 0$ is possible only for $\xi_1 \in \mathcal{S}_1$, where $\mathcal{S}_1 = \{\xi_1 : \|\xi_1\| \leq [\bar{c}_1 + \sqrt{(\bar{c}_1^2 + 4\bar{c}_2)}]/2\}$. Because $\mathcal{S}_1$ is compact and contains the point $\xi_1 = 0$, it follows that $\xi_1 \in \mathcal{L}^\infty$. □

## Appendix I: Adaptive Algorithms with Projection

Properties of such adaptive algorithms will be illustrated based on the example of a simple error model given here:

$$\dot{\boldsymbol{e}} = -\lambda \boldsymbol{e} + \phi\omega(t, \boldsymbol{x})$$

where $\lambda > 0$, $\boldsymbol{e} = \boldsymbol{x} - \boldsymbol{x}_m$, $\boldsymbol{x}_m : \mathbb{R}^+ \to \mathbb{R}$ is a smooth bounded function, $\omega(t, \boldsymbol{x})$ is bounded for bounded $\boldsymbol{x}$ and for all time, $\phi = \theta - \theta^*$ denotes the parameter error, $\theta$ is an adjustable parameter, and $\theta^* \in [\theta_{\min}, \theta_{\max}]$ is constant. The objective is to adjust $\theta(t)$ within $[\theta_{\min}, \theta_{\max}]$ so that $\lim_{t\to\infty} \boldsymbol{e}(t) = 0$.

*Theorem I1:* If $\theta(t)$ is adjusted using the adaptive algorithm with projection of the form

$$\dot{\theta} = \underset{[\theta_{\min}, \theta_{\max}]}{\mathrm{Proj}} \{-\gamma \boldsymbol{e}\omega\}, \qquad \theta(0) \in [\theta_{\min}, \theta_{\max}]$$

where the projection operator is defined as

$$\underset{[\theta_{\min}, \theta_{\max}]}{\mathrm{Proj}} \{-\gamma \boldsymbol{e}\omega\} = \begin{cases} -\gamma \boldsymbol{e}\omega, & \text{if } \theta(t) = \theta_{\max} \text{ and } \boldsymbol{e}\omega > 0 \\ 0, & \text{if } \theta(t) = \theta_{\max} \text{ and } \boldsymbol{e}\omega \leq 0 \\ -\gamma \boldsymbol{e}\omega, & \text{if } \theta_{\min} < \theta(t) < \theta_{\max} \\ 0, & \text{if } \theta(t) = \theta_{\min} \text{ and } \boldsymbol{e}\omega \geq 0 \\ -\gamma \boldsymbol{e}\omega, & \text{if } \theta(t) = \theta_{\min} \text{ and } \boldsymbol{e}\omega < 0 \end{cases} \tag{I1}$$

then $\lim_{t\to\infty} \boldsymbol{e}(t) = 0$.

*Proof:* Let the tentative Lyapunov function for the system be

$$V(\boldsymbol{e}, \phi) = \tfrac{1}{2}(\boldsymbol{e}^2 + \phi^2/\gamma) \tag{I2}$$

Its derivative along the motions of the system yields

$$\dot{V}(\boldsymbol{e}, \phi) = -\lambda \boldsymbol{e}^2 + \boldsymbol{e}\phi\omega + \phi\dot{\phi}/\gamma$$

To assure that $\dot{V}$ is negative semidefinite, our objective is to show that in all cases

$$\phi\dot{\phi} \leq -\gamma \boldsymbol{e}\phi\omega$$

that is, that $\phi\dot{\phi} + \gamma \boldsymbol{e}\phi\omega \leq 0$. We will consider each individual case further. We note that, because $\theta^*$ is constant, $\dot{\theta}(t) \equiv \dot{\phi}(t)$.

Case 1 is $\theta(t) = \theta_{\max}$. When $\boldsymbol{e}\omega > 0$, $\dot{\phi} = -\gamma \boldsymbol{e}\omega$ and $\phi\dot{\phi} = -\gamma \boldsymbol{e}\phi\omega$. Because $\phi = \theta - \theta^*$ and $\theta^* \in [\theta_{\min}, \theta_{\max}]$, in this case $\phi = \theta_{\max} - \theta^* \geq 0$. When $\boldsymbol{e}\omega \leq 0$, $\dot{\phi} = 0$, and we have $\phi\dot{\phi} + \gamma \boldsymbol{e}\phi\omega = \gamma \boldsymbol{e}\phi\omega \leq 0$.

Case 2 is $\theta_{\min} < \theta(t) < \theta_{\max}$. In this case $\phi\dot{\phi} = -\gamma \boldsymbol{e}\phi\omega$.

Case 3 is $\theta(t) = \theta_{\min}$. For $\boldsymbol{e}\omega < 0$, $\dot{\phi} = -\gamma \boldsymbol{e}\omega$ and $\phi\dot{\phi} = -\gamma \boldsymbol{e}\phi\omega$. Because, in this case, $\phi = \theta_{\min} - \theta^* \leq 0$ for $\boldsymbol{e}\omega \geq 0$, we have $\dot{\phi} = 0$ and $\phi\dot{\phi} + \gamma \boldsymbol{e}\phi\omega = \gamma \boldsymbol{e}\phi\omega \leq 0$.

It follows that the adaptive algorithms with projection assure that the condition $\phi\dot{\phi} \leq -\gamma \boldsymbol{e}\phi\omega$ is satisfied for all values of arguments. This implies that $\dot{V}(\boldsymbol{e}, \phi) \leq -\lambda \boldsymbol{e}^2 \leq 0$. Using the arguments from Appendix G, we can now readily demonstrate that $\lim_{t\to\infty} e(t) = 0$.

Our next objective is to demonstrate that when the adjustment law (I1) is used, the overall system will be robust to bounded external disturbances, time-varying parameters, and parametric uncertainty. Let the error model be of the form

$$\dot{\boldsymbol{e}} = -\lambda \boldsymbol{e} + \xi(t, \boldsymbol{x}) + \phi\omega(t, \boldsymbol{x}) + z$$

where $|\xi(t, \boldsymbol{x})| \leq a|\boldsymbol{x}| + b$ for all $(t, \boldsymbol{x})$ denotes a term due to unmodeled dynamics and/or parametric uncertainty, $a, b > 0$, and $a < \lambda$, $\theta^*(t) \in [\theta_{\min}, \theta_{\max}]$ for all $t$, $|\dot{\theta}^*(t)| \leq c_\theta$ for all $t$, and $|z(t)| \leq c_z$ for all $t$ denotes a disturbance. We also let $|\boldsymbol{x}_m(t)| \leq c_{xm}$ for all $t$.

*Theorem I2:* Adaptive algorithm (I1) assures that $\boldsymbol{e}$ is bounded.

*Proof:* Let the tentative Lyapunov function be of the form (I2). Because $\dot{\phi} = \dot{\theta} - \dot{\theta}^*$, its first derivative yields

$$\begin{aligned} \dot{V}(\boldsymbol{e}, \phi) &\leq -\lambda \boldsymbol{e}^2 + \boldsymbol{e}\xi + \boldsymbol{e}z - \phi\dot{\theta}^*/\gamma \\ &\leq -(\lambda - a)\boldsymbol{e}^2 + (ac_{xm} + b + c_z)|\boldsymbol{e}| + [(\theta_{\max} - \theta_{\min})c_\theta]/\gamma \end{aligned}$$

Because $\lambda > a$, using Appendix H, we can readily show that $\boldsymbol{e}$ is bounded, which completes the proof. □

## Acknowledgment

This research was supported by the Office of Naval Research under Contract N00014-98-M-0123 to Scientific Systems Co.

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